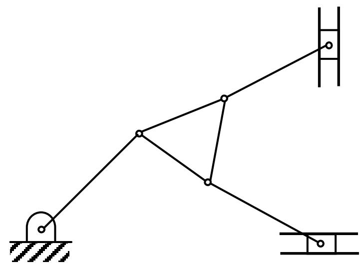
ESCUELA UNIVERSITARIA DE DISEÑO INDUSTRIAL

TEORÍA DE MÁQUINAS
(10 de diciembre de 2004)

Cuestiones:

1. Criterio de Grübler. Determinar el número de grados de libertad del siguiente mecanismo plano: (1 punto)

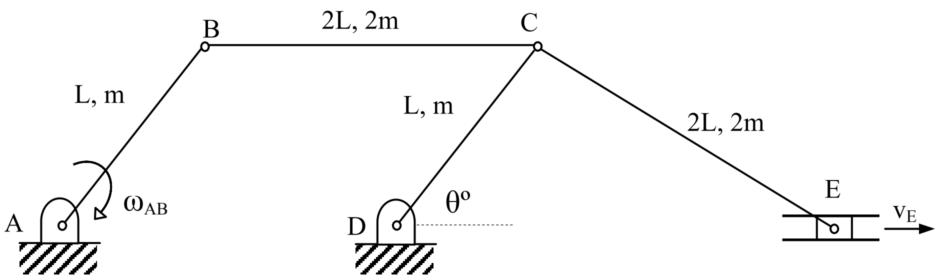


2. Elección de puntos de precisión para la generación de función mediante un cuadrilátero articulado. (1 punto)

Problemas:

1. La corredera E del mecanismo de la figura se mueve hacia la derecha con velocidad lineal uniforme v_E. Obtener, en función de la velocidad v_E: (3 puntos)

   i. Nº de grados de libertad del mecanismo.
   ii. Velocidad del punto C, v_C
   iii. Velocidad angular ω_{AB} de la barra AB.



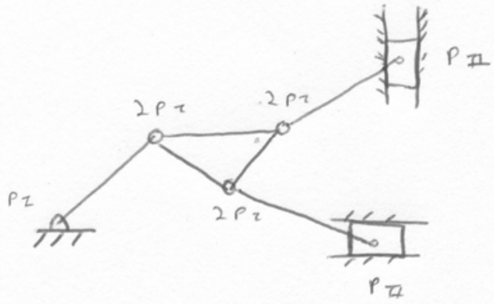
2. Dimensionar un cuadrilátero articulado tipo manivela-balancín, que cumpla las siguientes especificaciones: relación de tiempos Q=1,25, longitud del balancín r_4=18 cm, ángulo total de oscilación 80º, distribuido simétricamente con respecto a la vertical. (3 puntos)

3. Construir un engrane mediante dos ruedas cilíndrico-rectas con las siguientes especificaciones: (2 puntos)

Relación de transmisión μ=5.	Potencia a transmitir P=12 C.V.
Distancia entre ejes E=360 mm.	E acero=$2{,}1\cdot10^4$ Kg/mm².
n1=100 r.p.m.	$\sigma_{admisible}$= 300 Kg/cm².
Ángulo de presión ψ=20º.	Duración mínima = 10^5 horas.
β=10.	γ_c=9,62.

TIEMPO ESTIMADO 3 HORAS

1- CRITERIO DE GRÜBLER



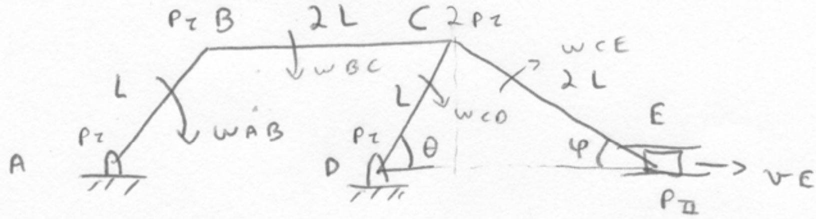
$n = 7$

$P_I = 7$

$P_{II} = 2$

$$GDL = 3(7-1) - 2 \cdot 7 - 2 = 18 - 14 - 2 = 2$$

1) i)



$n = 5$

$p_I = 5$

$p_{II} = 1$

$$GDL = 3(5-1) - 2\cdot 5 - 1 = 1 \quad GDL$$

ii)

$$\vec{v}_C = \vec{v}_E + \cancel{\vec{v}_r}^{\,0} + \vec{\omega}_{CE} \wedge \vec{EC}$$

$$\vec{EC} = +2L\left(-\cos\varphi\,\vec{i} + \operatorname{sen}\varphi\,\vec{j}\right)$$

$$L \operatorname{sen}\theta = 2L \operatorname{sen}\varphi \Rightarrow$$

$$\operatorname{sen}\varphi = \frac{\operatorname{sen}\theta}{2}$$

$$\cos\varphi = \sqrt{1-\operatorname{sen}^2\varphi} = \sqrt{1-\frac{\operatorname{sen}^2\theta}{2}}$$

$$\vec{w}_{CE} = -w_{CE}\,\vec{k}$$

$$\vec{w}_{CE} \wedge \vec{EC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -w_{CE} \\ -2L\cos\varphi & 2L\,\text{sen}\,\varphi & 0 \end{vmatrix} =$$

$$\vec{w}_{CE} \wedge \vec{EC} = 2L\,w_{CE}\,\text{sen}\,\varphi\,\vec{i} + 2L\,w_{CE}\cos\varphi\,\vec{j}$$

$$\vec{v}_E = v_E\,\vec{i}$$

$$\vec{v}_C = \left(v_E + 2L\,w_{CE}\,\text{sen}\,\varphi\right)\vec{i} + 2L\,w_{CE}\cos\varphi\,\vec{j}$$

$$\vec{v}_C = \cancel{\vec{v}_D} + \cancel{\vec{v}_r} + \vec{w}_{CD} \wedge \vec{DC}$$

$$\vec{DC} = L\cos\theta\,\vec{i} + L\,\text{sen}\,\theta\,\vec{j}$$

$$\vec{w}_{CD} \wedge \vec{DC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -w_{CD} \\ L\cos\theta & L\,\text{sen}\,\theta & 0 \end{vmatrix} =$$

$$\vec{w}_{CD} = -w_{CD}\,\vec{k}$$

$$\vec{v}_C = \vec{w}_{CD} \wedge \vec{DC} = L\,w_{CD}\,\text{sen}\,\theta\,\vec{i} - L\,w_{CD}\cos\theta\,\vec{j}$$

IGUALANDO -

$$v_E + 2L\,\omega_{CE}\,\text{sen}\,\varphi = L\,\omega_{CD}\,\text{sen}\,\theta$$

$$2L\,\omega_{CE}\cos\varphi = -\,L\,\omega_{CD}\cos\theta$$

$$\omega_{CE} = \frac{-\,\omega_{CD}\cos\theta}{2\cos\varphi}$$

$$\omega_{CE} = \frac{L\,\omega_{CD}\,\text{sen}\,\theta - v_E}{2L\,\text{sen}\,\varphi}$$

$$-\frac{\omega_{CD}\cos\theta}{\cancel{2}\cos\varphi} = \frac{L\,\omega_{CD}\,\text{sen}\,\theta - v_E}{\cancel{2}L\,\text{sen}\,\varphi}$$

$$-L\,\omega_{CD}\cos\theta\,\text{sen}\,\varphi = L\,\omega_{CD}\,\text{sen}\,\theta\cos\varphi - v_E\cos\varphi$$

$$\omega_{CD} = \frac{v_E\cos\varphi}{L\,(\cos\theta\,\text{sen}\,\varphi + \text{sen}\,\theta\cos\varphi)}$$

$$\text{sen}\,\varphi = \text{sen}\,\theta/2$$

$$\cos\varphi = \sqrt{1 - \frac{\text{sen}^2\theta}{2}}$$

$$w_{CD} = \frac{v_E \sqrt{1 - \frac{\operatorname{sen}^2\theta}{2}}}{L\left(\cos\theta \, \frac{\operatorname{sen}\theta}{2} + \operatorname{sen}\theta \sqrt{1 - \frac{\operatorname{sen}^2\theta}{2}}\right)}$$

$$\vec{v}_C = L \, w_{CD} \left(\operatorname{sen}\theta \, \vec{i} - \cos\theta \, \vec{j}\right) =$$

$$\vec{v}_C = \frac{\sqrt{1 - \frac{\operatorname{sen}^2\theta}{2}}}{\operatorname{sen}\theta} \left(\frac{\operatorname{sen}\theta \, \vec{i} - \cos\theta \, \vec{j}}{\frac{\cos\theta}{2} + \sqrt{1 - \frac{\operatorname{sen}^2\theta}{2}}}\right) v_E$$

iii) $\vec{v}_C = \vec{v}_B + \cancel{\vec{v}_r} + \vec{w}_{BC} \wedge \vec{BC}$

$$\vec{v}_B = \vec{v}_A + \vec{v}_r + \vec{w}_{AB} \wedge \vec{AB}$$

$$\vec{AB} = L\cos\theta \, \vec{i} + L\operatorname{sen}\theta \, \vec{j}$$

$$\vec{w}_{AB} = -w_{AB} \, \vec{k}$$

$$\vec{w}_{AB} \wedge \vec{AB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -w_{AB} \\ L\cos\theta & L\operatorname{sen}\theta & 0 \end{vmatrix} =$$

$$\vec{v}_B = \vec{\omega}_{AB} \wedge \vec{AB} = L\omega_{AB}\left(\operatorname{sen}\theta\,\vec{i} - \cos\theta\,\vec{j}\right)$$

$$\vec{\omega}_{BC} = -\omega_{BC}\,\vec{k}$$

$$\vec{BC} = +2L\,\vec{i}$$

$$\vec{\omega}_{BC} \wedge \vec{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -\omega_{BC} \\ 2L & 0 & 0 \end{vmatrix} =$$

$$\vec{\omega}_{BC} \wedge \vec{BC} = -2L\omega_{BC}\,\vec{i}$$

$$\vec{v}_c = \left(L\omega_{AB}\operatorname{sen}\theta - 2L\omega_{BC}\right)\vec{i} - L\omega_{AB}\cos\theta\,\vec{j}$$

$$\vec{v}_c = L\omega_{CD}\left(\operatorname{sen}\theta\,\vec{i} - \cos\theta\,\vec{j}\right) \Rightarrow$$

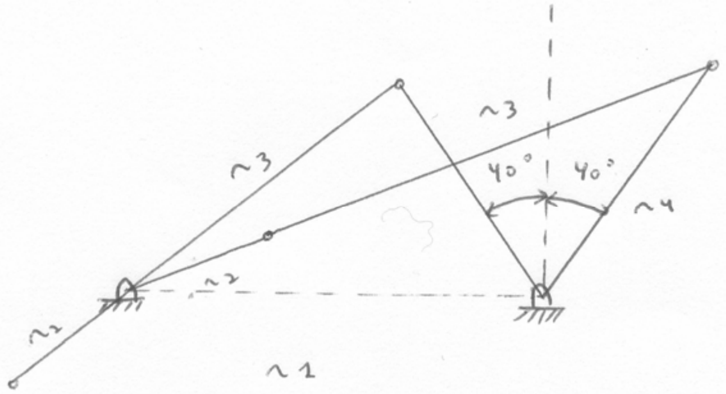
$$-L\omega_{AB}\cos\theta = -L\omega_{CD}\cos\theta \Rightarrow$$

$$\omega_{AB} = \omega_{CD}$$

$$L\omega_{AB}\operatorname{sen}\theta - 2L\omega_{BC} = L\omega_{CD}\operatorname{sen}\theta \Rightarrow$$

$$\omega_{BC} = 0$$

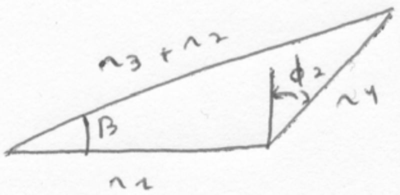
2°)



$$Q = \frac{180 + \alpha}{180 - \alpha} = 1{,}25 \Rightarrow \quad 180 + \alpha = 1{,}25\,(180 - \alpha)$$

$$(1 + 1{,}25)\,\alpha = 180\,(1{,}25 - 1)$$

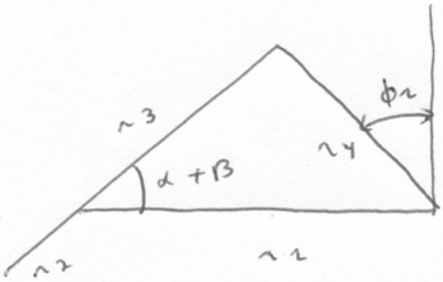
$$\alpha = \frac{0{,}25}{2{,}25} \cdot 180 = 20^\circ$$



$$(r_3 + r_2) \cos \beta = r_1 + r_4 \operatorname{sen} \phi_2$$

$$(r_3 + r_2) \operatorname{sen} \beta = r_4 \cos \phi_2$$

$$\operatorname{tg} \beta = \frac{\operatorname{sen} \beta}{\cos \beta} = \frac{r_4 \cos \phi_2}{r_1 + r_4 \operatorname{sen} \phi_2}$$



$$(r_3 - r_2)\cos(\alpha+\beta) + r_4 \operatorname{sen}\phi_1 = r_1$$

$$(r_3 - r_2)\operatorname{sen}(\alpha+\beta) = r_4 \cos\phi_1$$

$$(r_3 - r_2)\cos(\alpha+\beta) = r_1 - r_4 \operatorname{sen}\phi_1$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{sen}(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{r_4 \cos\phi_1}{r_1 - r_4 \operatorname{sen}\phi_1}$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha\,\operatorname{tg}\beta}$$

OPERANDO

$$0 = r_1^2 \operatorname{tg}\alpha + r_1 r_4 (\operatorname{tg}\alpha \operatorname{sen}\phi_2 + \cos\phi_2 - \operatorname{sen}\phi_1 \operatorname{tg}\alpha - \cos\phi_1) +$$

$$r_4^2 (\operatorname{tg}\alpha \cos\phi_1 \cos\phi_2 - \cos\phi_1 \operatorname{sen}\phi_2 - \operatorname{tg}\alpha \operatorname{sen}\phi_1 \operatorname{sen}\phi_2 - \operatorname{sen}\phi_1 \cos\phi_2)$$

$r_4 = 18$

$\alpha = 20°$

$\phi_1 = \phi_2 = 40$

$$0 = r_1^2 \, tg\,20 + 18\, r_1 \left(\cancel{tg\,20\ sen\,40} + \cancel{cos\,40} - \cancel{sen\,40\ tg\,20} - \cancel{cos\,40} \right) +$$

$$+ 18^2 \left(tg\,20\ cos\,40\ cos\,40 - cos\,40\ sen\,40 - tg\,20\ sen\,40\ sen\,40 + \right.$$

$$\left. - sen\,40\ cos\,40 \right)$$

$$0 = r_1^2 \, tg\,20 + 18^2 \left(tg\,20 \left(cos^2 40 - sen^2 40 \right) - 2\, sen\,40\, cos\,40 \right)$$

$$r_1^2 = \frac{18^2}{tg\,20} \left(2\, sen\,40\, cos\,40 - tg\,20 \left(cos^2 40 - sen^2 40 \right) \right)$$

$$r_1^2 = \frac{324}{0,3639} \left(2 \cdot 0,6428 \cdot 0,766 - 0,3639 \left(0,766^2 - 0,6428^2 \right) \right)$$

$$r_1^2 = 820,5 \Rightarrow r_1 = 28,64 \text{ cm}$$

$$tg\,\beta = \frac{r_4 \cos\phi_2}{r_1 + r_4\, sen\,\phi_2} = \frac{18\ cos\,40}{28,64 + 18\ sen\,40} =$$

$$tg\,\beta = 0,343 \Rightarrow \beta = 18,92^\circ \Rightarrow$$

$$r_3 + r_2 = \frac{r_4 \cos\phi_2}{sen\,\beta} = \frac{18\ cos\,40}{sen\ 18,92} = 42,51$$

$$r_3 - r_2 = \frac{r_4 \cos \phi_1}{\operatorname{sen}(\alpha + \beta)} = \frac{18 \cos 40}{\operatorname{sen}(20 + 18{,}92)} = 21{,}95$$

$$2 r_3 = 42{,}51 + 21{,}95 = 64{,}46$$

$$r_3 = 32{,}23 \text{ cm}$$

$$r_2 = 32{,}23 - 21{,}95 = 10{,}28 \text{ cm}$$

3) $\mu = 5 = \frac{n_2}{n_1} = \frac{z_1}{z_2} = \frac{R_1}{R_2} \Rightarrow$

$n_2 = \mu n_1 = 5 \cdot 100 = 500$ rpm

$R_1 = 5 R_2$

$E = R_1 + R_2 = 5R_2 + R_2 = 6 R_2 = 360 \Rightarrow$

$R_2 = \frac{360}{6} = 60$ mm

$R_1 = 5 R_2 = 5 \cdot 60 = 300$ mm

$m = \frac{2R}{z} \Rightarrow E = R_1 + R_2 = \frac{m}{2}(z_1 + z_2)$

$\mu = \frac{z_1}{z_2} = 5 \Rightarrow z_1 = 5 z_2$

$E = \frac{m}{2}(5z_2 + z_2) = 3 m z_2 = 360 \Rightarrow$

$z_2 = \frac{360}{3m} = \frac{120}{m}$

Si $m = 6 \Rightarrow$ $\left.\begin{array}{l} z_2 = \frac{120}{6} = 20 \\ z_1 = 5 z_2 = 100 \end{array}\right\}$ No hay penetración $z > z_{min} = 17$

COMPROBACIÓN DINÁMICA -

$$m \geq 35,7 \sqrt[3]{\frac{1000 \cdot P\,(CV)\, \gamma_c}{Z \cdot n\,(rpm)\, \sigma\,(Kg/cm^2)\, \beta}}$$

$$m \geq 35,7 \sqrt[3]{\frac{1000 \cdot 12 \cdot 9,62}{100 \cdot 100 \cdot 300 \cdot 10}} = 5,59$$

$m = 6$ valdría

$R_1 = 300$ mm

$R_2 = 60$ mm

$h_c = m = 6$ mm

$h_p = m + \frac{1}{6} m = \frac{7}{6} m = 7$ mm

$h = h_c + h_p = 6 + 7 = 13$ mm

$t = m \cdot \pi = 6 \cdot \pi = 18,85$ mm

$e = s = t/2 = 9,42$ mm

$b = \beta \cdot m = 10 \cdot 6 = 60$ mm

$$T = \frac{716200 \cdot P(CV)}{n(rpm) \cdot R(mm)} = \frac{716200 \cdot 12}{100 \cdot 300} =$$

$$T = 286,48 \text{ Kg}$$

$$N = \frac{T}{\cos\psi} = \frac{286,48}{\cos 20} = 304,865 \text{ Kg}$$

$$rm = \frac{2R_1 \operatorname{sen}\psi}{\mu + 1} = \frac{2 \cdot 300 \cdot \operatorname{sen} 20}{5 + 1} = 34,20 \text{ mm}$$

$$\sigma F = \sqrt{\frac{0,35 \cdot N \cdot E_a}{b \cdot rm}} =$$

$$\sigma F = \sqrt{\frac{0,35 \cdot 304,86 \cdot 2,1 \cdot 10^4}{60 \cdot 34,20}} = 33,04 \text{ Kg/mm}^2$$

$$a = \frac{n(rpm) \cdot h(\text{horas}) \cdot 60}{10^6} \text{ millones rev}$$

$$a = \frac{100 \cdot 100.000 \cdot 60}{10^6} = 600$$

$$\Delta 1 = \frac{1}{0,487} \, a^{1/6} \, \sigma F = \frac{1}{0,487} \, 600^{1/6} \cdot 33,04 =$$

$$\Delta 1 = 197,06 \quad HB$$

$$\Delta 2 = \sqrt[6]{\mu} \; \Delta 1 = \sqrt[6]{5} \; 197,06 = 257,69 \; HB$$