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AN EFFICIENT SIMULTANEOUS SOLUTION OF MULTIBODY SYSTEM DYNAMICS AND STRESS ANALYSIS FOR INTERACTIVE SIMULATION

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1 Introduction

Mechanical designers, concerned about the kinematics and dynamics of their part-mobile products, are increasingly using numerical codes for the dynamic simulation of multibody systems. However, very often they are also interested in the level of stress and strain incurred by their components. Automotive, aerospace, robotic and biomechanical industries are just some examples.
This work comes to efficiently an accurately solve this problem, allowing the designer to interactively visualize the fields of stresses and strains of multibody systems components mapped on their graphical rendering as motion takes place, as well as, of course, obtaining the exhaustive numerical information through plots or tables once the simulation has been performed. The combined use of fast and robust formulations for multibody dynamics with efficient methods for modeling flexible bodies, enables this involved problem to be solved with a similar accuracy to that provided by nonlinear FEA codes but spending far less time.

2 Background and contribution

There are two traditional ways to calculate the stresses suffered by components of a multibody system during its motion:
a) The first one consists of initially solving the motion of the mechanism through the use of a multibody dynamics code assuming that members are rigid. This provides the loads and reactions that serve as input for a FEA program, where the flexible condition of the elements is taken into account, to obtain stresses and strains. This approach is an approximation, since both the large amplitude motion and the elastic deformation are coupled and, therefore, they cannot be solved

separately. Moreover, data transference between programs becomes rather involved as long as formats are usually different.
b) The second one addresses the coupled problem making use of the nonlinear dynamics module of a FEA program, taking into account large displacements and finite rotations. This method, although theoretically correct, seems to be very slow, as it generates models with a large number of variables, leading to big problem sizes that require an enormous computational effort.

In this paper, a third way is proposed as an alternative: the dynamics of the multibody system are solved as rigid, but components whose stresses and strains are required, are modeled as flexible bodies. Indeed, any method of those developed to solve the dynamics of multibody systems with flexible bodies provides a relationship between the problem variables and the deformed configuration of each flexible link. Therefore, once positions are known at a certain point of time, local elastic displacements of the body can be directly calculated and strains and stresses immediately derived from them. Consequently, although this information is not explicitly obtained during a conventional dynamic analysis, it can be calculated with very little effort for each time-step of integration. In what follows, both the dynamic formulation and the modeling of flexible bodies used in this work are described.

The multibody dynamics of the proposed approach are solved by an improved version of the index-3 augmented Lagrangian formulation with mass-orthogonal projections given in [Bayo and Ledesma 1996]. The modeling, assumed in dependent coordinates, is carried out in natural coordinates [García de Jalón and Bayo 1994]. The integrator used is the implicit, single time-step trapezoidal rule. The integrator scheme is introduced in the dynamic equations, so leading to a non-linear algebraic set where the unknowns are the positions at the next time-step, which is solved by a Newton-Raphson iteration procedure. The quasi-tangent matrix is always positive-definite, so that robustness and a good convergence rate are guaranteed. Inherent desestabilization to any index-3 formulation is prevented by cleaning velocities and accelerations at each time-step by means of mass-orthogonal projections. The leading matrices of these projections and the quasi-tangent matrix of the Newton-Raphson iteration mentioned before are exactly the same. Hence, additional calculation effort due to projections is dramatically reduced as it only implies two forward reductions and back substitutions, as long as triangularization of the common leading matrix is already performed.

The modeling of flexible bodies is carried out by a moving frame approach with component mode synthesis for small elastic displacements. The global motion of each flexible body is described as a superposition of the large-amplitude motion of a moving frame, rigidly attached to a certain point of the body, and the small elastic displacements of the body with respect to an undeformed configuration, taken as reference. Any deformed configuration of the body is expressed as a linear combination of static and dynamic modes, in the sense of the mode synthesis approach with fixed boundaries. The static modes depend on the natural coordinates defined at the joints of the body while the number of internal, dynamic modes should be decided by the analyst. A detailed description of this method can be found in [Avello 1995] and [Cuadrado et al 1996].

3 Objective and examples

In this work, the problem of determining the level of stress undergone by the components of a mechanical system during its motion is adressed through two different methods:
a) A multibody system dynamic analysis, just considering the flexibility of those bodies whose stresses are of interest. This method will be referred to as MSD (Multibody System Dynamics). The corresponding code has been developed by the authors as an implementation of the proposed formulation, already described.
b) A dynamic analysis performed by a nonlinear module of a FEA program which takes into account large displacements and finite rotations. This method will be referenced as FEA (Finite Element Analysis). For this purpose, commercial code COSMOS/M 1.75A has been used.

The objective of this work is to demonstrate that the MSD method, proposed by the authors, enables to achieve a similar level of accuracy to that provided by the FEA method, while being largely more efficient. To this aim, comparison between both formulations in terms of accuracy and efficiency is established for two academic examples.

3.1 Flexible link with a bang-bang torque profile at the articulated joint

This first example, shown in Figure 1a, consists of a beam pinned at one end to the ground, which undergoes the torque depicted in Figure 1b. Gravity effects are neglected. Physical properties of the beam are: mass density 8000 Kg/m^3, modulus of elasticity $2x10^{11}$ N/m^2, length 1.5 m, cross-sectional area 10^{-4} m^2, moment of inertia 10^{-10} m^4.

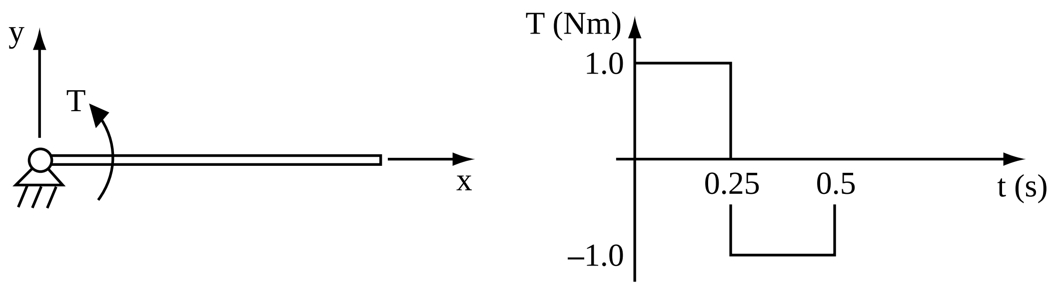

Figure 1. (a) Pinned-free beam. (b) Bang-bang torque.

For the MSD method, the modeling of the beam has been carried out as illustrated in Figure 2. At the pinned end, point p1 and unit vectors v1 and v2 are defined, thus constituting the local reference frame of the body. In this case, point p1 is fixed. At the free end, point p2 is defined, whose local displacement in v2-direction activates static mode Φ. To better represent the deformed configuration of the beam, dynamic modes Ψ_1 and Ψ_2 are also considered. They are the two first natural modes of vibration of the beam with fixed boundaries (points p1 and p2, and unit vectors v1 and v2), which means that, for their calculation, left end must be clamped and right end must be pinned.

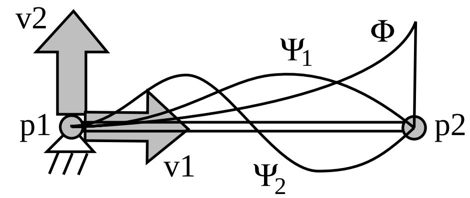

Figure 2. Modeling of a flexible pinned-free beam with the MSD method.

As consequence, the vector of problem variables results,

$$\mathbf{q}^{\mathrm{T}} = \{v1_x \quad v1_y \quad v2_x \quad v2_y \quad \eta \quad \xi_1 \quad \xi_2 \quad p2_x \quad p2_y\}$$

where η, ξ_1, ξ_2 are the amplitudes of the static and dynamics deformation modes, respectively. Therefore, the total number of variables is 9, with only 4 independent.

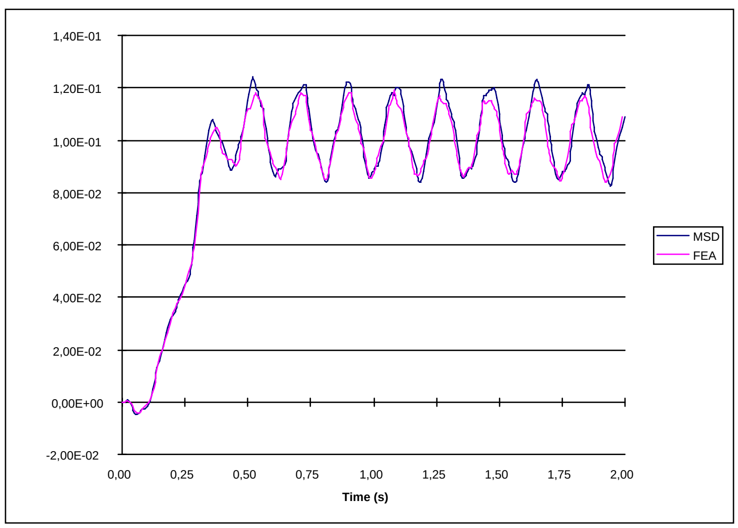

Figure 3. History of the vertical coordinate of the beam with MSD and FEA methods.

For the FEA method, the beam has been modeled by making a mesh of ten bidimensional beam elements (BEAM2D), all of them identical, with nodes of three degrees of freedom: two displacements in the plane of the beam and the corresponding slope. As the node placed at the pinned end of the beam can only experiment rotation, the total number of variables rises to 31 for this method.

A simulation of 2 seconds is performed through both methods. The time-step in both cases is 0.001 seconds. As a measurement of the motion obtained in each case, the history of the vertical coordinate of the free end of the beam is shown in Figure 3. Regarding the stress field of the beam, Figure 4 represents the history of the normal stress at the upper point of the middle section of the beam, also calculated with both methods.

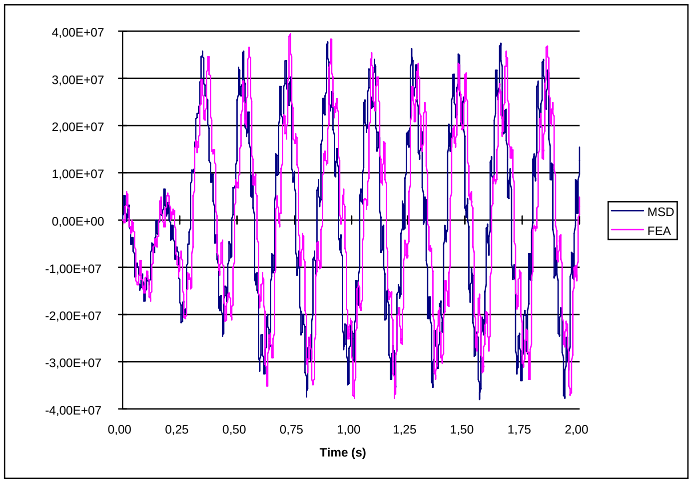

Figure 4. History of the maximum normal stress at the middle section of the beam with MSD and FEA methods.

Finally, Figure 5 shows the CPU times needed by each method to perform the simulation.

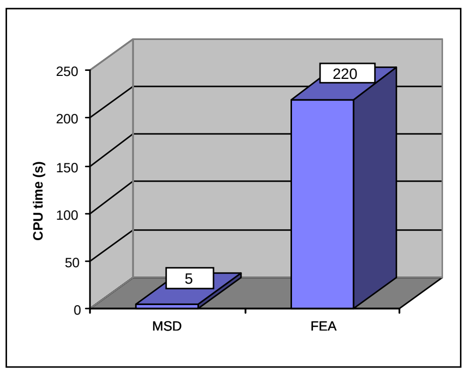

Figure 5. Efficiency of MSD and FEA methods.

3.2 Double-pendulum under gravity effects

The second example is a double-pendulum that starts from the rest in horizontal position, and falls under gravity action. Both links are identical and their physical properties are: mass density 2000 Kg/m^3, modulus of elasticity $7x10^{10}$ N/m^2, length 1.5 m, cross-sectional area $1.2x10^{-3}$ m^2, moment of inertia $4x10^{-7}$ m^4. In this case, the objective is to simulate 5 seconds of motion of the system, determining the stresses suffered by the first link along the time.

For the MSD method, the modeling of the double pendulum is illustrated in Figure 6. As only stresses at the first link are of interest, it is modeled as flexible, while the second link is modeled as rigid. Modeling of the flexible body is identical to that described in the previous example.

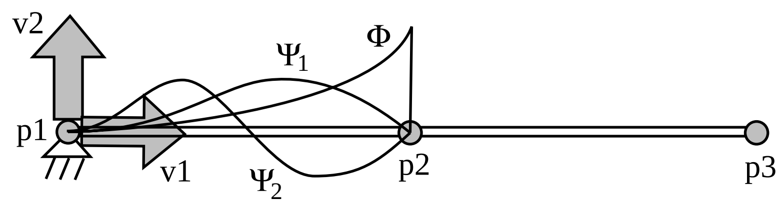

Figure 6. Modeling of a double pendulum with the MSD method.

Hence, the vector of problem variables is the following,

$$\mathbf{q}^{\mathrm{T}} = \{v1_x \quad v1_y \quad v2_x \quad v2_y \quad \eta \quad \xi_1 \quad \xi_2 \quad p2_x \quad p2_y \quad p3_x \quad p3_y\}$$

which means that the total number of variables is 11, with only 5 independent.

For the FEA method, each link has been modeled in an identical way of that explained above for the previous example, imposing in this case the identity of nodal displacements at the connection between both links. Therefore, the total number of variables is 62.

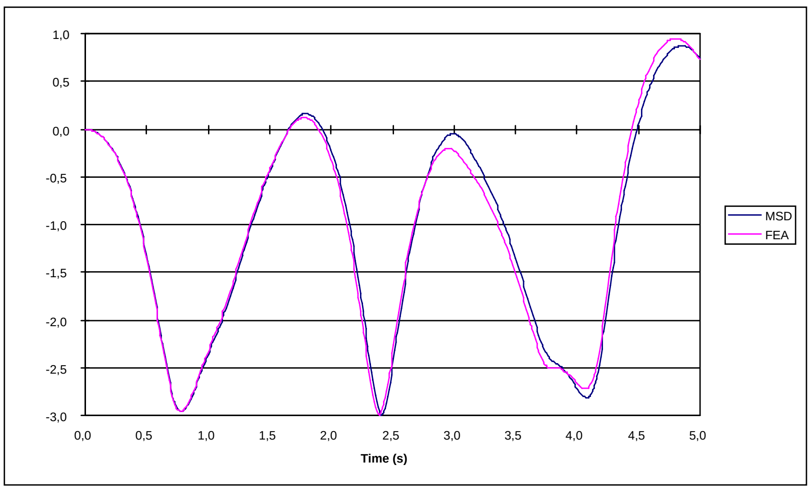

Figure 7. History of the vertical coordinate of the end of a double-pendulum with MSD and FEA methods.

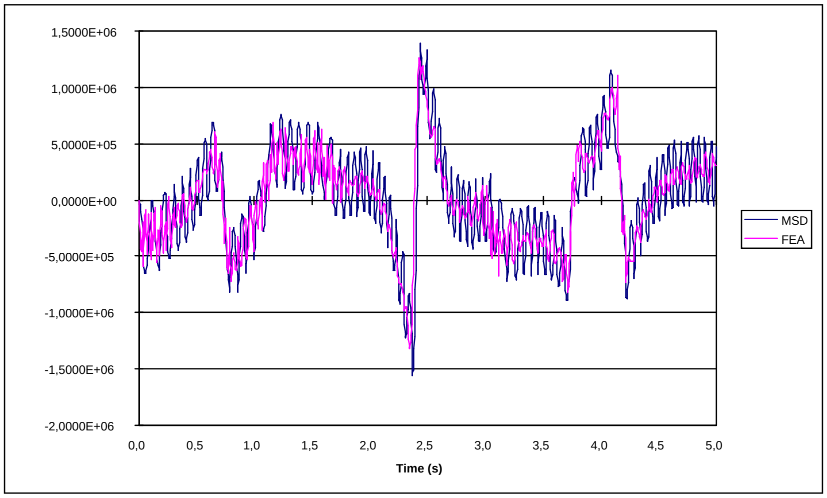

Figure 8. Maximum normal stress at the middle section of the first link of a double-pendulum.

The simulation of 5 seconds is carried out through both methods using a time-step of 0.001 seconds. Figure 7 shows the history of the vertical coordinate of the free end of the double-pendulum. In Figure 8, the maximum normal stress at the middle section of the first link due to bending effects is represented.

To finish, Figure 9 shows the CPU times spent by both methods in performing the simulation.

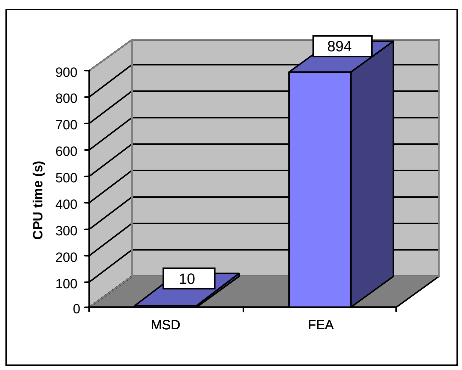

Figure 9. Efficiency of the MSD and FEA methods.

All the calculations for both examples have been performed on a SGI Indigo2 IMPACT with one processor R4400SC @ 200 MHz and 2 Mb of secondary cache memory.

4 Conclusions

Based on the obtained results, the following conclusions may be established:

• For similar accuracy, the proposed MSD method is largely more efficient than the FEA method, near two orders of magnitude faster, almost achieving real-time performance.

• The aforementioned trend is increased when the number of bodies of the mechanical system grows, as the difference in problem size between both methods goes up for more complex and sophisticated systems.

Acknowledgements

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