Efficiency of a Semi-Recursive Penalty Formulation when Applied to Flexible Multibody Systems

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Keywords: efficiency, real-time, flexibility, semi-recursive, penalty, FFR.

1. Introduction

Efficiency is an issue for any application in multibody dynamics, and becomes essential for those requiring real-time performance, like human- and hardware-in-the-loop simulations.

In a previous work [1], the authors developed a semi-recursive penalty formulation which integrates in time the relative coordinates of the mechanism, dependent in the general case (presence of closed loops). The formulation was compared with two others: a global penalty formulation which integrates the natural coordinates of the system (dependent), and a semi-recursive formulation which integrates a minimum set of coordinates (independent). The comparison showed that the proposed semi-recursive penalty formulation achieves the best results in terms of efficiency and robustness for large rigid multibody systems.

The present work reports on the study being conducted in order to evaluate the behavior of the mentioned formulation when dealing with flexible multibody systems. The objective is to find out whether the advantages shown by such formulation in the rigid case are kept when flexible bodies are also considered. Since, in the flexible case, the minimum number of coordinates is usually much higher than in its rigid counterpart, the third formulation mentioned above, i.e. that which integrates a set of independent coordinates only, has not been included in the study. Therefore, the comparison is established between the semi-recursive penalty formulation integrating relative coordinates, and the global penalty formulation integrating natural coordinates. In both cases, the flexible bodies are modeled by means of the floating frame of reference method (FFR), according to the form proposed by the authors in [2].

2. Semi-recursive penalty formulation

For rigid multibody systems, the first step consists of applying the cut-joint method, so as to convert the multibody system into its open-tree version, and describing its motion through the corresponding relative coordinates. Then, the dynamic equations are stated according to an index-3 augmented Lagrangian formulation in the form,

$$\begin{aligned} &\mathbf{M}\ddot{\mathbf{z}} + \mathbf{\Phi}_{\mathbf{z}}^T \alpha \mathbf{\Phi}_{\mathbf{z}} + \mathbf{\Phi}_{\mathbf{z}}^T \boldsymbol{\lambda}^* = \mathbf{Q} \\ &\boldsymbol{\lambda}_{i+1}^* = \boldsymbol{\lambda}_i^* + \alpha \mathbf{\Phi}_{i+1}, \quad i = 0,1,2,... \end{aligned} \tag{1}$$

where $\mathbf{z}$ are the relative coordinates, $\mathbf{M}$ is the mass matrix of the mechanism expressed in terms of the relative coordinates, $\mathbf{\Phi}$ is the constraints vector due to the closure conditions of the loops, $\mathbf{\Phi}_{\mathbf{z}}$ is the Jacobian matrix of the constraints, α is the penalty factor, $\mathbf{Q}$ is the vector of applied and velocity-dependent forces, and $\boldsymbol{\lambda}^*$ is the vector of Lagrange multipliers.

In order to determine the dynamic terms $\mathbf{M}$ and $\mathbf{Q}$, a Cartesian set of coordinates is defined for each body of the mechanism,

$$\mathbf{Z}^T = \{\dot{\mathbf{s}} \quad \boldsymbol{\omega}\} \tag{2}$$

being $\dot{s}$ the velocity of the point of the body which at that particular time is coincident with the fixed frame origin, and $\boldsymbol{\omega}$ the angular velocity of the body.

A matrix $\mathbf{R}$ can be defined so that the following relationship stands,

$$\mathbf{Z} = \mathbf{R}\dot{\mathbf{z}} \tag{3}$$

where now **Z** includes the Cartesian coordinates (2) of all the bodies of the mechanism. Due to the use of such coordinates, matrix **R** can be written as,

$$\mathbf{R} = \mathbf{T}\mathbf{R}_d \tag{4}$$

with **T** a connectivity matrix, and $\mathbf{R}_d$ a block-diagonal matrix. Due to the especial structure of matrix **R**, both matrix **M** and vector **Q** of the whole mechanism, required in (1), are easily obtained through and efficient recursive procedure, from the corresponding terms at body level expressed in the Cartesian coordinates (2).

On the other hand, the constraints needed to close the loops can be established in natural coordinates with very little effort, the Jacobian being,

$$\mathbf{\Phi}_\mathbf{z} = \mathbf{\Phi}_\mathbf{q}\mathbf{q}_\mathbf{z} \tag{5}$$

where $\mathbf{\Phi}_\mathbf{q}$ is the traditional Jacobian matrix of the constraints when natural coordinates are used, and $\mathbf{q}_\mathbf{z}$ represents the velocities of the natural coordinates **q** when unit velocities are successively given to the relative coordinates **z**.

3. Consideration of flexible bodies

The FFR method proposed by the authors in [2] employs natural coordinates to describe each flexible body. It defines the local reference frame of the body with a point $\mathbf{r}_0$ at the origin, and a set of orthogonal unit vectors **u**, **v** and **w** for the local axes. Then, the position of an arbitrary point **r** of the body can be defined as,

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{A}\cdot\overline{\mathbf{r}} \tag{6}$$

being **A** the rotation matrix, whose columns are the three unit vectors mentioned, and $\overline{\mathbf{r}}$ the deformed position of the point in local coordinates. In order to increase the efficiency, this deformed position is expressed by modal superposition,

$$\overline{\mathbf{r}} = \overline{\mathbf{r}}_u + \sum_{i=1}^{ns}\boldsymbol{\varphi}_i\eta_i + \sum_{j=1}^{nd}\boldsymbol{\psi}_j\xi_j \tag{7}$$

where $\overline{\mathbf{r}}_u$ is the undeformed position, $\boldsymbol{\varphi}_i$ and $\boldsymbol{\psi}_j$ are the *nd* static and *ns* dynamic mode shapes, and η_i and ξ_j are their respective amplitudes. The static modes depend on the points and unit vectors defined at the joints of the body. Then, a flexible body is described by the local frame ($\mathbf{r}_0$, **u**, **v**, **w**) and the modal amplitudes (η_i, ξ_j), and the mass matrix and forces vector are obtained with respect to such variables.

Consequently, for these flexible bodies to be included into the semi-recursive penalty formulation presented above, the following set of coordinates should be defined at body level,

$$\mathbf{Z}^T = \left\{\dot{\mathbf{s}} \quad \boldsymbol{\omega} \quad \eta_1 \quad \cdots \quad \eta_{ns} \quad \xi_1 \quad \cdots \quad \xi_{nd}\right\} \tag{8}$$

so that the mass matrix and forces vector are transformed to such new coordinates. Moreover, the set of relative coordinates describing the open-tree version of the rigid mechanism must be increased with the amplitudes of the static modes of all the flexible bodies considered. Provided the mentioned measures are taken, the recursive procedure to calculate the dynamic terms **M** and **Q** required in (1), already explained for rigid bodies, can be extended to the flexible case.

4. Comparison between formulations

In order to compare the behavior of both the global and semi-recursive formulations, the same examples already used for the rigid case in [1] will be addressed now. The strategy will consist of progressively substituting rigid bodies by flexible bodies, so that criteria may be established on which formulation to use, depending on the modeling conditions.

5. References

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