Improved efficiency in FFR methods for flexible multibody dynamics by means of shape integrals preprocessing

U. Lugrís*, J. Cuadrado

*** Escuela Politécnica Superior, University of La Coruña**
Mendizábal, s/n, 15403 Ferrol, Spain
Phone: 981337400; e-mail: ulugris@udc.es

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1. Introduction

In flexible multibody dynamics, one of the most widely used methods for considering flexible bodies is the Floating Frame of Reference (FFR) [1]. This method attaches a local frame of reference to each elastic solid, in such a way that the frame undergoes the large rigid body motion, and the elastic displacements are superimposed using local coordinates. The most common way to model the local deformation is by means of the finite element method, so that a reduction procedure, such as component mode synthesis, can be then applied to reduce the number of system coordinates.

In a previous work [2], the authors compare two FFR formulations, one based on absolute natural coordinates [3] and other using relative coordinates, concluding that the absolute method is better suited for small sized problems, whereas the relative method is faster when the system has a large number of coordinates.

In both cases, the co-rotational approximation proposed by Géradin and Cardona [4] is used to obtain the mass matrix and the velocity dependent inertia forces of flexible bodies. These terms are obtained at each time-step by performing matrix products in which the finite element mass matrix is involved. This implies that although the use of component mode synthesis reduces the number of coordinates to be integrated in time, the method does not take full advantage of the reduction, since large matrices of the finite element mode size still appear in the mass matrix and inertia forces calculation.

The main idea of this work is to study the performance of both the absolute and the relative formulations, when these terms are obtained by means of smaller matrix operations, involving matrices of the size of the reduced model. This is achieved by calculating several constant matrices, obtained through shape integrals in a preprocessing stage, as shown in [1]. Once the method is defined, a new comparison between the two formulations will be carried out, in order to check if the performance differences are kept or new criteria should be established.

2. Description of the method

The procedure will be described for the absolute formulation, being analogous for the relative one. The position of any given point $\mathbf{r}$ of a flexible body can be obtained as,

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{A}\overline{\mathbf{r}} = \mathbf{r}_0 + \mathbf{A}\left(\overline{\mathbf{r}}_u + \delta\overline{\mathbf{r}}\right) \tag{1}$$

where $\mathbf{r}_0$ stands for the position of the origin of the local frame of reference, $\mathbf{A}$ is a rotation matrix defined by the three orthogonal unit vectors of the reference frame [$\mathbf{u}$|$\mathbf{v}$|$\mathbf{w}$], $\overline{\mathbf{r}}_u$ is the undeformed position of the point in local coordinates, and $\delta\overline{\mathbf{r}}$ is its local elastic displacement. The elastic displacement at a point is approximated using component mode synthesis, which can be written in matrix form,

$$\delta\overline{\mathbf{r}} = \mathbf{X}\mathbf{y} \tag{2}$$

being $\mathbf{X}$ a matrix containing the mode shapes as columns, and $\mathbf{y}$ the vector of modal amplitudes. The body motion can be defined by the vector of variables of the body $\mathbf{q}$, which contains the position of the origin $\mathbf{r}_0$, the local frame vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, and the modal amplitudes $\mathbf{y}$,

$$\mathbf{q}^T = \left\{\mathbf{r}_0^T \quad \mathbf{u}^T \quad \mathbf{v}^T \quad \mathbf{w}^T \quad \mathbf{y}^T\right\} \tag{3}$$

By time differentiation of (1), a linear relationship can be established between the velocity of a point $\dot{\mathbf{r}}$ and the time derivatives of the body coordinates $\dot{\mathbf{q}}$,

$$\dot{\mathbf{r}} = \begin{bmatrix} \mathbf{I}_3 & \bar{x}\mathbf{I}_3 & \bar{y}\mathbf{I}_3 & \bar{z}\mathbf{I}_3 & \mathbf{AX} \end{bmatrix} \dot{\mathbf{q}} = \mathbf{B}\dot{\mathbf{q}} \tag{4}$$

where $\bar{x}$, $\bar{y}$ and $\bar{z}$ are the components of the deformed local position of the point $\bar{\mathbf{r}}$. This relationship can be introduced into the kinetic energy expression, to obtain the mass matrix $\mathbf{M}$,

$$T = \frac{1}{2}\int_V \dot{\mathbf{r}}^T \dot{\mathbf{r}}\, dm = \frac{1}{2}\int_V \dot{\mathbf{q}}^T \mathbf{B}^T \mathbf{B} \dot{\mathbf{q}}\, dm \quad \Rightarrow \quad \mathbf{M} = \int_V \mathbf{B}^T \mathbf{B}\, dm \tag{5}$$

which, after developing the matrix product $\mathbf{B}^T\mathbf{B}$, has the following form,

$$\mathbf{M} = \int_V \begin{bmatrix} \mathbf{I}_3 & \bar{x}\mathbf{I}_3 & \bar{y}\mathbf{I}_3 & \bar{z}\mathbf{I}_3 & \mathbf{AX} \\ & \bar{x}^2\mathbf{I}_3 & \overline{xy}\mathbf{I}_3 & \overline{xz}\mathbf{I}_3 & \bar{x}\mathbf{AX} \\ & & \bar{y}^2\mathbf{I}_3 & \overline{yz}\mathbf{I}_3 & \bar{y}\mathbf{AX} \\ & sym. & & \bar{z}^2\mathbf{I}_3 & \bar{z}\mathbf{AX} \\ & & & & \mathbf{X}^T\mathbf{X} \end{bmatrix} dm \tag{6}$$

where all the terms can be manipulated to put the body coordinates $\mathbf{A}$ and $\mathbf{y}$ outside the integrals. For example, the integral of $\bar{x}$,

$$\int_V \bar{x} dm = \int_V \left(\bar{x}_u + \mathbf{X}_x \mathbf{y} \right) dm = m\bar{x}_u^G + \left(\int_v \mathbf{X}_x dm \right) \mathbf{y} = m\bar{x}^G \tag{7}$$

In this expression, $\bar{x}_u$ is the undeformed local x coordinate, $\mathbf{X}_x$ is the first row of $\mathbf{X}$, m is the mass of the solid, and $\bar{x}_u^G$, $\bar{x}^G$ are the local x coordinates of the center of mass in the undeformed and deformed configurations respectively.

A similar procedure can be applied to all the remaining terms. All of them can be obtained from matrix operations involving the modal amplitudes $\mathbf{y}$, and constant matrices resulting from integrals of the mode shapes, the undeformed positions, and products between them. In the case of the relative formulation, the $\mathbf{B}$ matrix relates different coordinates to the nodal velocities, and the expressions are somewhat more complicated, but the procedure for obtaining the mass matrix is analogous.

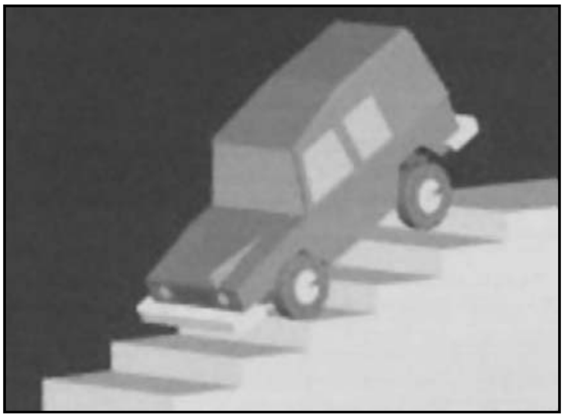

Fig. 1. Iltis maneuver.

As it was done in [2], a comparison between both formulations will be carried out by simulating different systems, such as the Iltis vehicle running through a rough profile (Fig. 1), and criteria will be established to decide which formulation should be used depending on the problem size and the number of elastic bodies.

Preliminary tests have shown that, in the absolute method, with flexible bodies modeled using 10 elements (33 DOFs) and 6 modes, performance can be increased about 175% by using preprocessing. In the case of large finite element models, the difference can be much more significant, since preprocessing makes the CPU-time independent of the finite element mesh size.

3. References

[1] A.A. Shabana, *Dynamics of Multibody Systems*, 2nd edition, Cambridge University Press, Cambridge, (1998).

[2] U. Lugrís, J. Cuadrado, F. González, A. Luaces, “Efficiency of topological and global formulations for small and large flexible multibody systems”, *Multibody Dynamics 2007, ECCOMAS Thematic Conference*, Proceedings CD, Milano, (2007).

[3] J. Cuadrado, R. Gutiérrez, M.A. Naya, P. Morer, “A Comparison in terms of accuracy and efficiency between a MBS dynamic formulation with stress analysis and a non-linear FEA code”, *International Journal for Numerical Methods in Engineering*, Vol. (51.9), 1033–1052, (2001).

[4] M. Géradin and A. Cardona, *Flexible Multibody Dynamics – A Finite Element Approach*, John Wiley and Sons, New York, (2001).