Proceedings of MUSME 2008, the International Symposium
on Multibody Systems and Mechatronics
San Juan (Argentina), 8-12 April 2008
Paper n. 02-MUSME08

Optimization of an off-road bicycle with four-bar linkage rear suspension

M. González, J. Cuadrado, F. González and D. Dopico
Laboratory of Mechanical Engineering, University of A Coruña
Mendizábal s/n, 15403 Ferrol, Spain
E-mail: lolo@cdf.udc.es Web: http://lim.ii.udc.es

Abstract: Despite full suspension bicycles provide a physiological and psychological advantage over the hard tail bicycles in bumpy terrains, they may also dissipate the cyclist's energy through small oscillatory movements of the rear suspension, often termed 'bobbing'. Some authors have proposed optimization techniques to minimize the energy dissipation due to bobbing, but they only consider the single-pivot suspension design, which has been superseded by the four-bar linkage design. This paper presents enhanced dynamic model of an off-road bicycle, which is used to optimize a four-bar linkage rear suspension.

Keywords: bicycle, suspension, energy efficiency, dynamic simulation, optimization

INTRODUCTION

Bicycle suspensions have become quite popular in off-road biking, since they improve comfort, handling, braking and line holding by dissipating terrain-induced energy [Olsen (1996)]. Most of the bicycle suspension systems are designed to act on the front wheel or on the rear wheel. Front suspension (FS) systems are widely used in off-road bicycles, since they have no disadvantages except for a slight weight penalty in the fork; bicycles that only use front suspension are also referred as hard tail bicycles. Conversely, full dual suspension (DS) bicycles, which have both front and rear suspension systems, are not broadly accepted by off-road racers. Despite Titlestad et al 2006 demonstrated that full suspension bicycles provide a physiological and psychological advantage over the hard tail bicycles in a bumpy treadmill, they may also dissipate the cyclist's energy through small oscillatory movements of the rear suspension, often termed 'bobbing'. According to Nielens and Lejeune (2004), bobbing can be generated by two mechanisms: (i) the frame induced loads due to the periodic displacement of the rider’s body; and (ii) the interaction between the suspension and the forces transmitted by the chain to the rear wheel. Wang and Hull (1996) measured this power dissipation for a seated cyclist riding a commercial full suspension bicycle up a 6% grade at 6.5 m/s, resulting in a 1.3% of the total power input by the rider: for these conditions, the bicycle appeared to be 12% heavier than it actually was. Moreover, in standing position the power dissipation is almost one order of magnitude higher, as measured by Karching and Hull (2002). This phenomenon is a major concern for off-road cross-country competitive cyclists, which spend most of the time in climbing, and justifies their preference for hard tail bicycles.

In order to minimize the power dissipation of rear suspensions when climbing on smooth surfaces, major manufacturers have proposed new designs in the last decade. Nielens and Lejeune (2004) reviewed the most relevant studies conducted to measure their energy efficiency, and concluded that there is not enough evidence to make a decision about the use of hard tail or full suspension bicycles: the type and degree of suspension and its effect on performance are topics of much debate yet, and further research about the subject is needed to objectively asses the benefits and drawback of different designs.

Energy efficiency can be evaluated by three methods: (i) to measure the metabolic expenditure of the cyclist in laboratory or field experiments; (ii) to measure the mechanical power dissipated by the suspension in laboratory or field experiments; and (iii) to measure the mechanical power dissipated by the suspension in computer simulations. The first two methods are experimental, which makes extremely difficult to analyze the dependence of the energy looses on the various system parameters, since it would require modifying the bicycle design. Needle and Hull (1997) partially achieved this by constructing an adjustable dual suspension off-road bicycle; despite its ingenious design, only a rear suspension type is contemplated (single pivot), and few parameters can be changed. As an alternative to experimental methods, computer simulations use a *dynamic model of the bicycle and rider*, making easier to gain insight about the dependence of the power dissipation on the various system parameters: bicycle geometry, suspension design, rider's anthropometric data and terrain profile can be easily modified and the corresponding energy efficiency can be evaluated in few seconds. These methods allows to optimize a large number of parameters in DS bicycles, or to select the best suspension design for a racer in a particular off-road cross-country competition.

Despite the advantages of computer simulations, only two dynamics models of a suspension bicycle and rider have been proposed by now. The first one was proposed by Wang and Hull (1996): this two-dimensional model simulated a seated rider pedaling up a constant grade of 6% at a constant velocity of 6.5 m/s on a smooth surface. Inertial and geometric parameter values for the bicycle were measured experimentally from a commercially available dual suspension bicycle with a swing arm type rear suspension, and anthropometric and inertial data for the rider were found using photographs and regression equations. The dynamic model was very detailed, including rolling resistance, wind resistance and tire flexibility. Results were validated experimentally, and the only major discrepancy between the experiments and the simulations was the presence of a phase lag in the suspension displacement, which was attributed to inter-subject variability. Later, the same model was used by Wang and Hull (1997) to optimize the pivot point location in terms of energy efficiency. Their results agreed to the experimental results obtained by Needle and Hull (1997) using an adjustable dual suspension off-road bicycle.

The dynamic model proposed by Wang and Hull (1996) shows a behavior similar to experimental results, but has a important disadvantage: the rider induced frame loads were applied as external loads to the handle-bar, seat and bottom bracket. The applied load data were acquired by Stone (1990) during a seated climbing on a 6% grade at 7.2 m/s using a road bicycle, since no data exists for off-road bicycles. Given that rider induced frame loads depend on bicycle geometry, suspension design and terrain grade, the load data obtained by Stone are only valid for simulations with a similar bicycle geometry and terrain grade; in other cases, the induced frame loads would have a different distribution.

In order to overcome the need to specify rider-bicycle interface loads, Good and McPhee (1999) developed a simplified four-body model using the same bicycle and rider data as Wang and Hull (1996): by including the body of the rider in the system model, the need to include rider-bike interface load is eliminated. Moreover, the front suspension was not modeled due to its small energy dissipation, and the only external applied forces in the model are the gravitational loads and a chain tension that varies with the crank angle with a harmonic function fitted to experimental data from Wang and Hull (1996). Despite its simplicity, this model produces results very similar to the model from Wang and Hull (1996), in terms of suspension displacements as a function of crank angle. Good and McPhee (2000) used this model to optimize the suspension design using a genetic algorithm.

While the model proposed by Good and McPhee (1999) represents a important contribution in the simulation of seated cyclists riding dual suspension bicycles, it does not consider the motion of the rider's body, one of the causes of bobbing. To justify this simplification, they calculated the vertical acceleration of the riders center of mass during cycling, resulting in $\pm$ 1 m/s^2, which means that the inertial loads derived from the rider's body movement are around 10% of the gravitational loads; however, they also recognize that it is difficult to know whether these inertial loads are significant. Nielens and Lejeune (2004) found that the effect of body motion on the energy dissipated by the suspension has never been studied in detail. This effect should not be neglected, because it is the single cause of bobbing in some recent rear suspension designs like the four-bar linkage or the unified rear triangle concept, which partially or totally eliminate the interaction between the suspension and the chain tension, as described by Nielens and Lejeune (2004). In these suspension designs, the model proposed by Good and McPhee (1999) would not cause bobbing. In addition, the effect of body motion may be significantly higher in standing position or sprinting.

Another limitation of the previous studies related to off-road bicycle suspension optimization is the type of rear suspension considered. Both Wang and Hull (1997), Needle and Hull (1997) and Good and McPhee (2000) optimized the design of a traditional single-pivot rear suspension, also known as swing-arm suspension. This type of rear suspension design has been superseded by the abovementioned modern designs like the four-bar linkage or the unified rear triangle concept (Figure 1). In fact, the four-bar linkage is one the most used rear suspension designs in racing off-road bicycles.

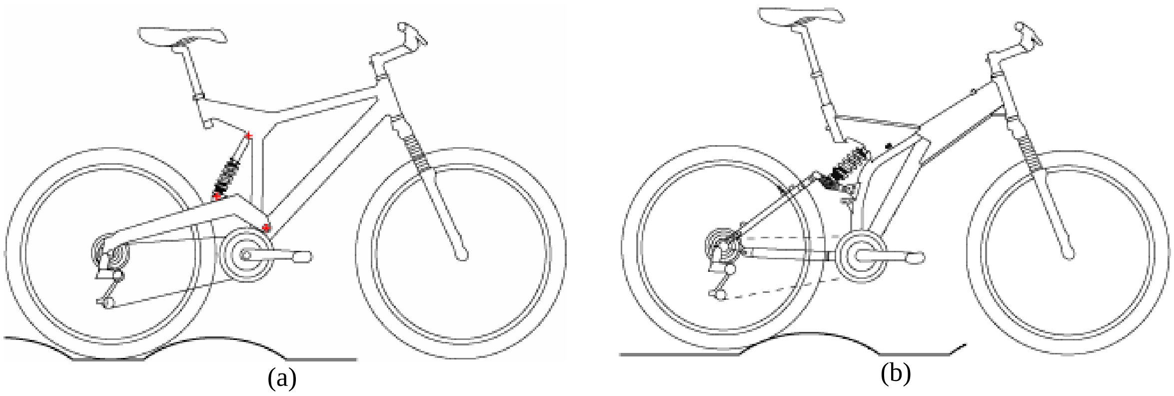

Figure 1. Two designs for rear suspensions in off-road bicycles: (a) single pivot; (b) four-bar linkage.

The current study has two goals: first, to investigate the effect of rider's body motion on the dynamics of dual suspension bicycles and its impact of the energy efficiency; and second, to optimize the design of a four-bar linkage rear suspension based on energy dissipation. The remainder of the paper is organized as follows: Section 2 investigates the effect of body motion in the dynamics of rear suspensions; Section 3 present the optimization process of a four-bar linkage rear suspension design; finally Section 4 provides conclusions and areas of future work.

EFFECT OF BODY MOTION IN THE SUSPENSION DYNAMICS

Rider and bicycle model

As stated in the Introduction, the dynamic model proposed by Good and McPhee (1999) does not consider the motion of the rider's body, one of the causes of bobbing. In order to quantify its effect, a new dynamics model has been developed, introducing two improvements:

- Good and McPhee (1999) applied the chain tension in a fixed direction. This is a simplification, since its direction varies due to the oscillatory movement of the rear suspension, and changes the effective torque transmitted to the rear wheel. In the proposed model, the exact direction of the chain tension is evaluated during the simulation.

- Good and McPhee (1999) introduced the rider's body as a rigid body fixed to the bicycle frame: the complete model had only four bodies. In the proposed model, only the upper part of the body (torso, head, arms and hands) is fixed to the frame, while thighs, shanks and cranks are introduced as separate bodies to allow leg motion. With this approach the resulting model has ten bodies, as shown in Figure 2

The system has been modeled using two-dimensional natural coordinates: Table 1 shows the points associated with each body. In addition to the fifteen points (30 coordinates), four relevant angles and distances are introduced as variables in the model: front and rear wheel angles, the crank angle θ, and the distance s between points 8 and 9, where the spring-damper element of the rear suspension is mounted. Therefore, the total number of coordinates in the model is 34. Constraint equations are derived from the rigid body condition of each element in the model, the definition of angles and distances, the rolling condition between the wheels and the road, and the gear joint between the sprockets. The resulting model has two degrees of freedom: the horizontal displacement, which can be measured by x_2, and the suspension displacement, which can be measured by the distance s.

Table 1. Bodies and natural coordinates used to model the system.

Body no.	Description	Reference points
1	Front wheel	1, 2, 3
2	Frame, fork and rider's upper body	2, 7, 9, 10, 13
3	Rear wheel	4, 5, 6
4	Right crank	10, 11
5	Right shank	11, 12
6	Right thigh	12, 13
7	Left crank	10, 14
8	Left shank	14, 15
9	Left thigh	13, 15
10	Rear triangle	5, 7, 8

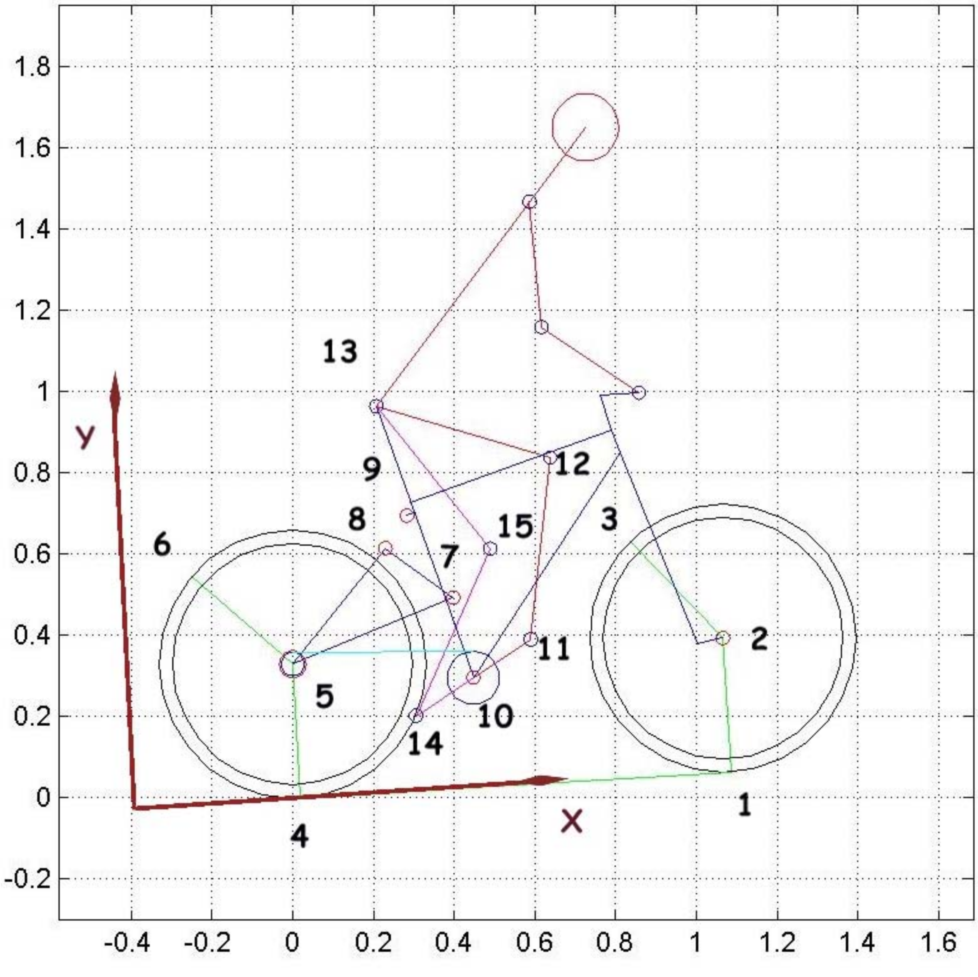

Figure 2. Dynamic model of the rider and single-pivot suspension bicycle showing the points used as coordinates.

Simulation conditions and dynamic formulation

In order to allow easy comparison with previous results, the simulation conditions used by Wang and Hull (1996) and Good and McPhee (1999) were also used in this work: a seated rider pedaling up a constant grade of 6% at a constant velocity of 6.5 m/s on a smooth surface, with the bicycle in a fixed gear combination of 38x14; the corresponding cadence was 84 rpm. To achieve a constant velocity, a kinematical constraint is included for the crank angle θ.

The magnitude of the chain tension was taken from the model by Good and McPhee (1999), as an harmonic function of the crank angle θ:

$$T = 42 \cdot (1 - \cos 2\theta) \tag{1}$$

The dynamic equations are formulated in independent coordinates using the projection matrix **R**, and combined with the trapezoidal rule as the numerical integration scheme [García de Jalón and Bayo (1994)]. The simulation code was implemented in the Matlab computation environment.

Dynamic simulation

To validate the proposed model, the dynamic simulation was run with the crank angle fixed, to neglect the effect of the ride's body motion. The final time of the simulation has been adjusted to allow four cycles in the movement of the crank movement, and only results from the last cycle are recorded. Results of this simulation were compared with the results from Wang and Hull (1996) and Good and McPhee (1999), as shown in Table 2 and Figure 3.

Table 2. Suspension displacement as a function of crank angle: correlation with published results (error is measured with respect to experimental results).

Results	Mean		Amplitude		Phase lag		Power dissipation
	mm	*% error*	mm	*% error*	Degrees	*error*	W
Experimental	-6.68		5.55		72º		
Wang and Hull (1996)							
Simulation							
Wang and Hull (1996)	-7.15	*7%*	6.05	*9%*	95º	*23º*	6.9
Good and McPhee (1996)	-7.54	*13%*	5.00	*10%*	98º	*26º*	-
Proposed model	-6.61	*1%*	7.61	*37%*	91º	*19º*	10.3

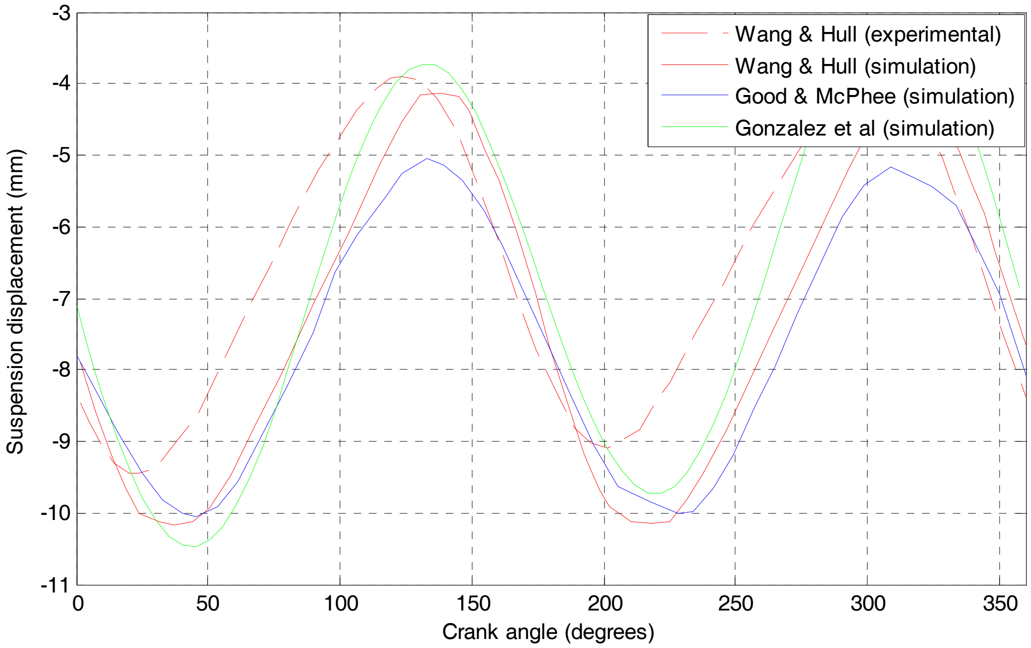

Figure 3 Suspension displacement as a function of crank angle: correlation with published results.

Table 3 compares the results generated by the fixed-leg model with the moving-leg model. The rider's body motion causes a higher compression in the spring-damper element of the suspension, but reduces the amplitude of its movement, and therefore reduces the power dissipation.

Table 3. Suspension displacement as a function of crank angle: correlation with published results (error is measured with respect to experimental results).

Results	Mean (mm)	Amplitude (mm)	Power dissipation (W)
Without leg motion	-6.61	7.61	10.3
With leg motion	-7.28	6.04	8.0

The influence of road grade and rider's pedaling cadence on energy dissipation was also investigated. It was found that the effect of road grade is minimal, but energy dissipation is indirect proportional to cadence, as shown in Table 4:

Table 4. Effect of pedaling cadence on power dissipation.

Cadence (r.p.m.)	Power dissipation (W)
58	11.1
71	9.7
84	8.0
97	6.4
110	5.0

The effect of the maximum crank torque generated by the cyclist on the energy dissipation, see Table 5, is also very important, as it was suspected:

Table 5. Effect of maximum crank torque on power dissipation.

Torque (Nm)		Power dissipation (W)	
34	(-20%)	4,95	(-38%)
38	(-10%)	6,37	(-20%)
42	(reference)	7,97	(reference)
46	(+10%)	9,76	(+22%)
50	(+20%)	11,74	(+47%)
63	(+50%)	19,46	(+144%)
84	(+100%)	36,15	(+354%)

The proposed dynamic model of the rider and dual suspension bicycle demonstrates that the inclusion of rider's body motion has a significant effect on the power dissipation in the suspension, since it is reduced in a 20 % compared with models which include the rider as a rigid body.

OPTIMIZATION OF A FOUR-BAR LINKAGE REAR SUSPENSION

Rider and bicycle model

A second dynamic model has been developed for a rider cycling on a bicycle with a four-bar linkage rear suspension. The model is shown in Figure 4 and Table 6 shows the points associated with each body. The resulting model has 12 bodies, 40 coordinates and two degrees of freedom: the horizontal displacement, which can be measured by x_2, and the suspension displacement, which can be measured by the distance *s*. In this case, the spring-damper element is located between points 9 and 18.

Table 6. Bodies and natural coordinates used to model the bicycle with four-bar linkage rear suspension.

Body no.	Description	Reference points
1	Front wheel	1, 2, 3
2	Frame, fork and rider's upper body	2, 9, 10, 13, 16, 17
3	Rear wheel	4, 5, 6
4	Right crank	10, 11
5	Right shank	11, 12
6	Right thigh	12, 13
7	Left crank	10, 14
8	Left shank	14, 15
9	Left thigh	13, 15
10	Rear triangle	5, 7, 8
11	Horst-link bar	7, 16
12	Spring-damper triangle	8, 17, 18

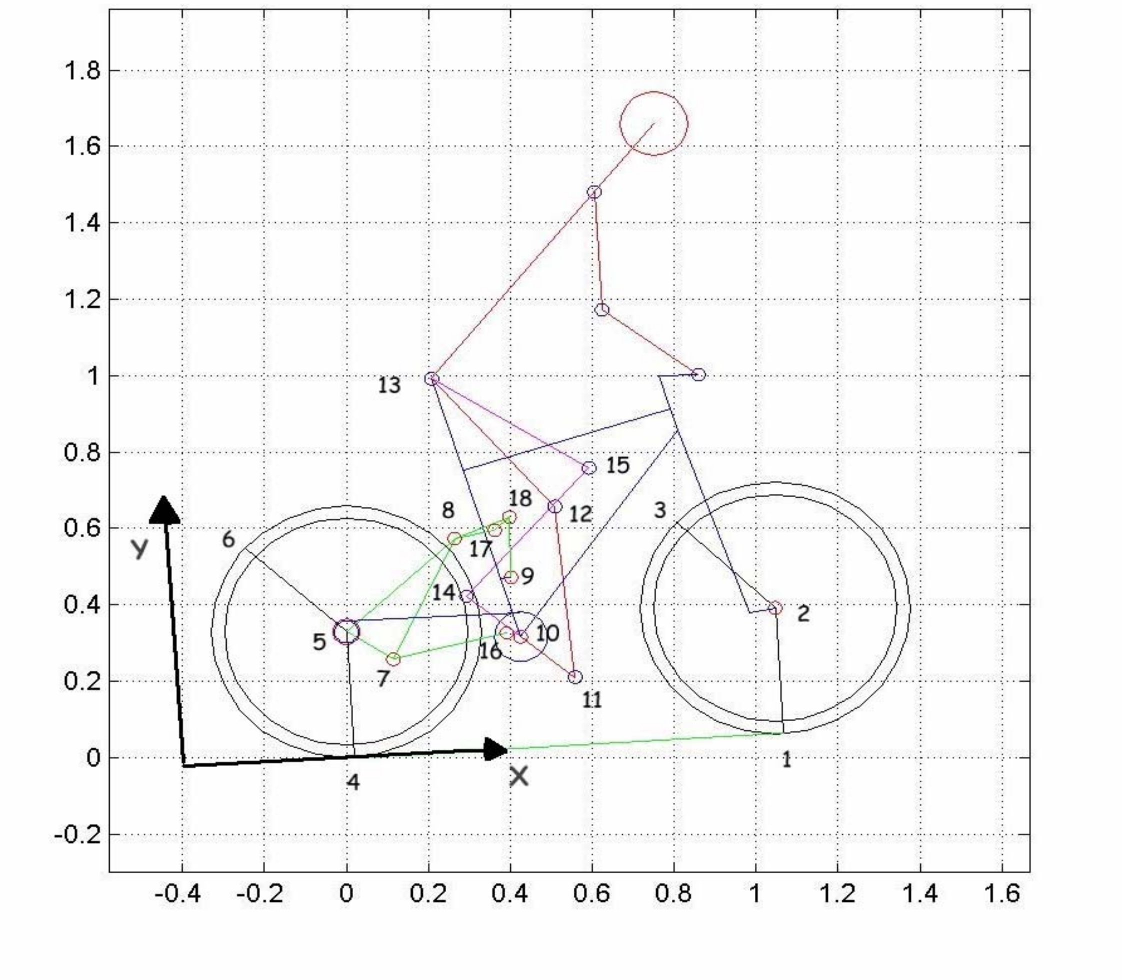

Figure 4. Dynamic model of the rider and four-bar linkage suspension bicycle.

Optimization method and results

In addition to the dynamic simulation of the bicycle and rider movement, the model was used to perform an optimization analysis to minimize the energy dissipated by the suspension. The design variables in the optimization are shown in Table 7. Appropriate constraints were included to avoid non-feasible designs and to preserve the main geometric parameters of the original bicycle (distance between front and rear wheels, distance from crack to ground, etc.). The optimization process was carried out with the Optimization Toolbox of Matlab.

Table 7. Design variables in the optimization process.

Design variable	Initial value (m)	Lower limit (m)	Upper limit (m)
x_{9L}	0,045	0,025	0,055
y_{9L}	0,080	0,075	0,095
x_{16L}	0,000	-0,030	0,000
y_{16L}	0,070	0,000	0,100
x_{17L}	-0,020	-0,030	0,020
y_{17L}	0,250	0,220	0,260
L_{716}	0,373	0,365	0,385
L_{57}	0,045	0,045	0,045
L_{58}	0,371	0,371	0,371
L_{78}	0,345	0,345	0,345
$L_{8\text{-}17}$	0,115	0,115	0,115
$L_{8\text{-}18}$	0,170	0,170	0,170
$L_{17\text{-}18}$	0,075	0,075	0,075

Figure 5 shows the evolution of the power dissipation during the optimization process. The final value is 0.80 W, an important reduction compared with the initial 10.8 W. In addition, the optimum design does not increase the total weight of the bicycle, as shown in Figure 4: the rear triangle is only 20 grams heavier after the optimization.

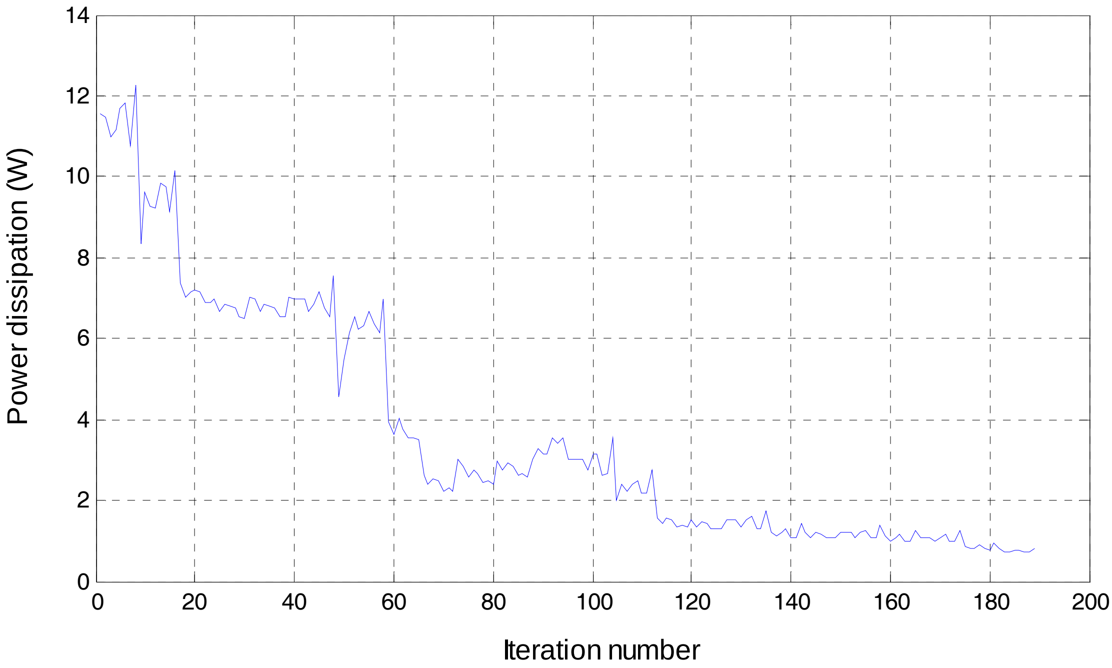

Figure 5. Evolution of the power dissipation in the optimization process.

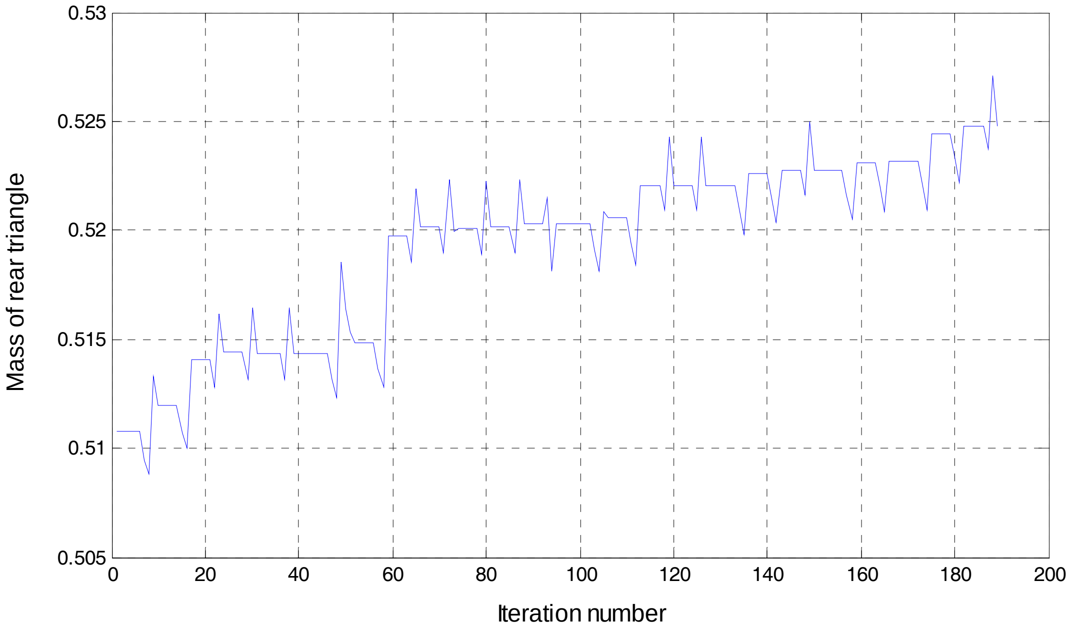

Figure 6. Evolution of the mass of the rear triangle in the optimization process..

The optimization process was run again using the fixed legs model, and the resulting optimum design was very similar to the optimum design of the moving legs model: the difference in power dissipation was around 20%, but its magnitude (less than 0.1 W) is too low to affect rider's performance.

CONCLUSIONS

The proposed dynamic model of the rider and dual suspension bicycle demonstrates that the inclusion of rider's body motion has a significant effect on the power dissipation in the suspension, since it is reduced in a 20 % compared with models which include the rider as a rigid body. However, the optimum suspension designs found with both models are very similar. In addition, the dependence of the power dissipation with respect to several variables related to the rider's motion was investigated. It was found that the power dissipation does not depend on the road grade, which goes against common belief, but increases heavily when the maximum crank torque generated by the cyclist increases.

The dynamic model was used to optimize the design of a four-bar linkage rear suspension, the most popular suspension design in off-road racing bicycles. The optimization process lead to a design with a significantly lower energy dissipation due to the bobbing effect. The same process can be used to improve other modern suspension designs for off-road bicycles.

References

García de Jalón, J. and Bayo, E (1994), "Kinematic and dynamic simulation of multibody systems – The real-time challenge". Springer-Verlag, New York.

Good, C. and McPhee, J. (1999). "Dynamics of Mountain Bicycles With Rear Suspensions: Modeling and Simulation," *Sports Engineering*, 2, pp. 129-143.

Good, C. and McPhee, J. (2000) "Dynamics of mountain bicycles with rear suspensions: design optimization". *Sports Engineering*, 3, pp. 49-55.

Karchin, A. and Hull, M. L. (2002). "Experimental Optimization of Pivot Point Height for Swing-Arm Type Rear Suspensions in Off-Road Bicycles". *ASME. Journal of Biomechanical Engineering.* Vol. 124, pp. 101-106.

Needle, S. and Hull, M. L. (1997). "An Off-Road Bicycle With Adjustable Suspension Kinematics". *ASME. J. Mech. Des*., 119, pp. 370-375.

Olsen, J. (1996). “Bicycle Suspension Systems”. *High- Tech Cycling*. pp. 45-64. Human Kinetics, Champaign, IL, USA.

Stone, C. (1990). “Bicycle Interaction Loads During Seated And Standing Treadmill Cycling”. MSc Thesis. Department of Mechanical Engineering, University of California at Davis. CA, USA

Stone, C. and Hull, M.L. (1995) The Effect of Rider Weight on Rider-Induced Loads During Common Cycling Situations”. *Journal of Biomechanics*, 28, 365-375.

Wang, E. L and Hull, M. L. (1996). "A Model for Determining Rider Induced Energy Losses in Bicycle Suspension Systems”. *Vehicle System Dynamics*, 25, pp. 223- 246.

Wang, E. L. and Hull, M. L. (1997). "Minimization of Peda1ing Induced Energy Losses in Off-Road Bicycle Rear Suspension Systems”. *Vehicle System Dynamics*, 28, pp. 291-306.