Use of Penalty Formulations in the Dynamic Simulation of Redundantly Constrained Multibody Systems

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Abstract

Redundantly constrained multibody systems have more constraints than required to ensure kinematically correct motion. This results in dependence among the constraint equations. Under these conditions, if the rigid body model is employed to represent the links of the system, the constraint forces cannot be fully determined in forward dynamics simulation [3]. In the real system, there is an intrinsic relationship between the forces and the deformations of the bodies, which fully specifies the reaction forces, even in the presence of redundant constraints. This relation is lost if the rigid body model is used. In this case, the equations of motion for a system defined by a set of n generalized velocities $\mathbf{v}$ and subject to m constraints $\mathbf{\Phi} = \mathbf{0}$ can be expressed as

$$\begin{bmatrix} \mathbf{M} & -\mathbf{A}^{\mathrm{T}} \\ -\mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_a - \mathbf{c} \\ \dot{\mathbf{A}}\mathbf{v} + \dot{\mathbf{b}} \end{bmatrix} \tag{1}$$

with $\mathbf{M}$ the mass matrix, $\mathbf{A}$ the constraint Jacobian matrix with respect to the generalized coordinates $\mathbf{q}$, $\boldsymbol{\lambda}$ the vector of Lagrange multipliers, $\mathbf{f}_a$ the vector of applied forces, $\mathbf{c}$ the vector of Coriolis and centrifugal forces, and $\mathbf{b} = \partial\mathbf{\Phi}/\partial t$. In the presence of redundant constraints, the accelerations $\dot{\mathbf{v}}$ and the generalized constraint forces, $\mathbf{A}^{\mathrm{T}}\boldsymbol{\lambda}$, are uniquely defined, provided that $\mathbf{M}$ and $\mathbf{\Phi}$ have been adequately given. The term $\mathbf{A}^{\mathrm{T}}\boldsymbol{\lambda}$ is the representation of the constraint forces in terms of the selected generalized coordinates and it represents the resultant effect of the constraints on the system. However, the particular reaction forces, as given by the Lagrange multipliers $\boldsymbol{\lambda}$, are not uniquely determined [1]. In fact, there is an infinite set of solutions for $\boldsymbol{\lambda}$, that corresponds to the set of feasible reaction forces compatible with the motion of the system.

Determining the motion alone is the primary goal of many dynamic simulations in which the precise determination of particular reaction forces is not required, only their resultant effect is needed. In these cases, it is enough for the simulation algorithm to compute the accelerations and the resultant of the constraint forces. This can be done with the use of reduction techniques [2]. However, if the reaction forces are of interest (e.g., when friction forces enter the picture), then their numerical value needs to be computed to obtain a realistic solution. In the general case, this cannot be done without dropping the rigid body assumption and including additional information about the structural properties of the system. A possible solution is to use flexible bodies to represent all the components of the mechanism, or at least those involved in the computation of the reaction forces that are not uniquely determined [4].

In some cases, penalty factors can be used as an alternative to flexible body modelling of the components of the mechanism, in conjunction with natural coordinates. Natural coordinates model each body in the mechanism with a set of points and vectors, bound together by a set of kinematic constraint equations representing the rigid body assumption. It is possible to relax these rigid body constraints by associating a penalty factor to each of them. It is not uncommon in the literature to assign a single penalty factor α to all the constraint equations. The penalty factor is then considered just as an 'arbitrarily large number'. However, an adequate scaling of the penalty factors $\alpha_k = \eta_k \alpha$ for each constraint equation can be used to approximate the structural properties of the mechanism, and to obtain accurate values of the reaction forces during the simulation in an efficient way.

As an example, we employed scaled penalty factors to approximate the structural properties of a spatial parallelogram mechanism (Figure 1, a) described in [4]. An augmented Lagrangian formulation of index-3 with projection of velocities and accelerations was used to solve Equation (1). The computed reaction forces (Figure 1, b) match those obtained via the use of flexible body models to represent the rods and plate of the mechanism.

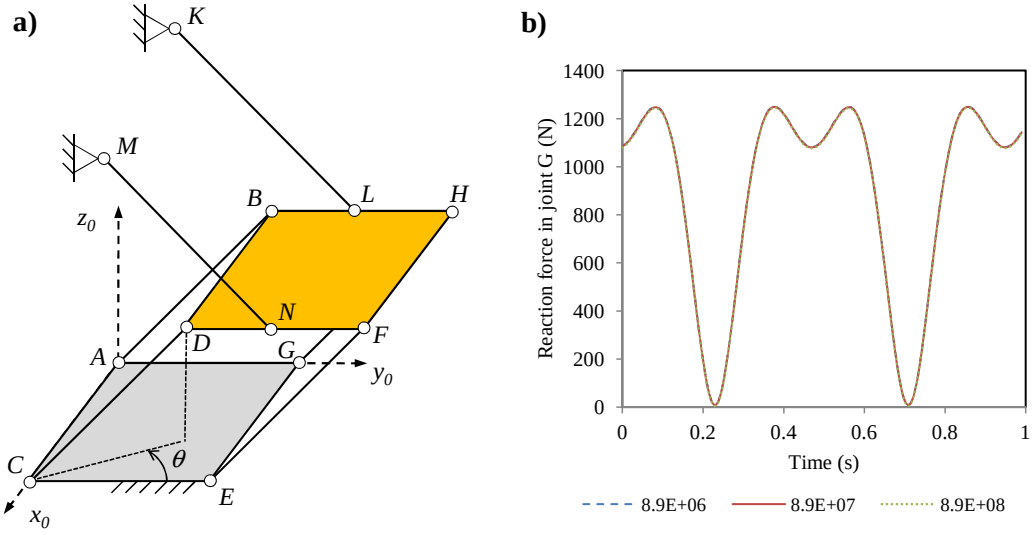

Figure 1: Spatial parallelogram (left) and reaction forces in joint G during motion of the parallelogram, for different values of factor α (right)

It was found that the scaling relation between the different penalty factors is of critical importance for the accuracy of the results, whereas the numerical value of the penalty factor α has a much less noticeable impact on them. Our analysis shows that the use of penalty factors can represent an efficient and easy way to determine constraint reaction forces, as an alternative to employing flexible multibody models. We believe that this is an important conclusion, which has not yet been shown in the literature. It should be stressed that a meaningful relation between the structural properties of the bodies and the definition of the constraint equations (which, in turn, determine the physical meaning of the penalty factors) has to be found for the modelling of the system. Once this relation is adequately defined, the value of the penalty factors can be adjusted to model the stiffness distribution in the system, which is dominant for the development of the constraint reactions.

References

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