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Use of Sub-System Global Modal Parameterization Models in Extended Kalman Filtering for Online Coupled State/Force Estimation.

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Abstract

This paper discusses the use of Sub-System Global Modal Parameterization (SS-GMP) reduced multibody models in an augmented discrete extended Kalman filter (A-DEKF) to generate a general formalism for online coupled state/input estimation in mechanisms. The SS-GMP approach is proposed to reduce a general multibody model of a mechanical system into a real-time capable model without considerable loss in accuracy. An exponential integration scheme is used to discretize the model in order to be compatible with discrete time filters. Finally, the augmented approach is used for the estimation, in which the unknown external forces are considered as additional states to be estimated. The proposed approach is validated numerically and compared to three other filtering approaches. The validation demonstrates that the proposed approach provides accurate results while still maintaining real-time performance.

Keywords: *Model reduction, Global Modal Parameterization, Kalman filter, input estimation*

1 Introduction

In many mechanical applications, there is a large need to know the current state of the system: for control purposes it is crucial to take appropriate action, for health monitoring the condition of the system has to be known, ...The most straightforward approach to this problem is the direct measurement of the state of interest. Quite often however, this is not feasible. Measurement locations can be hard to reach, sensors can be prohibitively expensive or no sensor might even exist for the variable of interest. In order to address this issue, state estimators have made a huge rise over the last decades. This rise was kickstarted by the introduction of the Kalman filter [1], and many variations of this approach have been presented over the years [2]. The main strength of this method is that it optimally leverages the a-priori knowledge of a system, by the use of a model, and the measurements.

State-estimators generally require an accurate model in order to provide good results. Multibody simulation provides a general framework to develop high fidelity models for mechanical systems [3]. Ideally these models would also be applied for state-estimation [4, 5]. Unfortunately this last approach leads to models which cannot be run in real-time together with an extended Kalman filter for systems with multiple degrees-of-freedom (DOFs). In order to obtain real-time capable multibody models, this paper proposes the Sub-System Global Modal Parameterization (SS-GMP) [6]. This method is a system-level model reduction technique for multibody systems. Whereas the original GMP approach [7, 8] did not perform well in the case of multiple DOFs, SS-GMP allows the division of a mechanism into multiple submodels which can be efficiently reduced by GMP and can then be connected back together [9]. In this work the SS-GMP approach is proposed for systems consisting of rigid bodies and localized force elements. A brief summary of the methodology is provided in Sec. 2.

The second issue for many mechanical systems is the fact that the input-forces are not known. Regular state estimators assume that the inputs to the system are known, but in practice this is rarely the case. This typically leads to bias-errors on the state estimates and degraded performance. In mechanical systems it is often of special interest to obtain an estimate of the external loads to a system as well. This is clearly the case in vehicle applications where the friction coefficient [10, 11] and vertical displacement of the road is usually unknown but crucial for proper control. However this issue can be extended to many other domains like mechatronics [12, 13, 14] and biomechanics. Several variations of the regular Kalman filter have been proposed to perform combined state and input estimation [15, 16, 17]. These approaches can be categorized under *unknown input observers* [18] and have the advantage of providing a simultaneous estimation of states and forces.

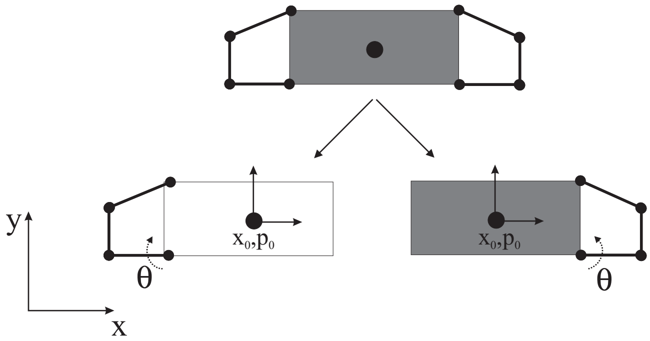


Figure 1: Reduced coordinates for a SS-GMP

In this work, the augmented Kalman filter, which has been applied with success to linear [19] and nonlinear [10] mechanical systems, is chosen in order to take the unknown inputs into account [15]. The augmented approach is applied to an extended Kalman filter due to the nonlinear equations of motion for the multibody system. Moreover, in order to maximize real-time performance the filter is described in the discrete time domain, which is more suitable for efficient implementation [6]. The resultant filtering approach is an *augmented discrete extended Kalman filter* (A-DEKF). The algorithm for this filter is discussed in Sec. 3. In order to obtain discrete time equations from the continuous time equations of motion for the multibody system, an exponential discretization [20] is applied.

Finally the proposed approach is validated numerically in Sec. 4. The proposed A-DEKF approach coupled with an SS-GMP model is shown to be real-time capable through an implementation in FORTRAN.

2 Connected Sub-System Global Modal Parameterization

In this work the Sub-System Global Modal Parameterization (SS-GMP) approach is proposed as model reduction method to create models for mechanical systems for use in state estimators. This approach was first introduced in the frame of real-time simulation of flexible multibody systems [9] and is applied to rigid mechanisms in this work.

In this section, equations are presented for the planar case for the sake of clarity, but results can be easily extended to the spatial case as presented by Naets [9] for flexible systems.

The SS-GMP modeling approach consists of two steps [9]:

- *Preprocessing*: during this phase, the model is split up into sub-models which can be reduced separately. Subsequently the reduction, according to the GMP approach [7, 8], is performed.

- *Processing*: during the simulation, the equations of motion for each sub-model are evaluated based on the interpolation of the stored system matrices and the equations presented hereafter. After each sub-model is evaluated, the redundant DOFs are eliminated and the equations of motion for the full system are evaluated.

2.1 Dividing model into sub-models

The SS-GMP is developed specifically for mechanical systems which are the assembly of multiple sub-mechanisms with closed kinematic loops (Fig. 1). The SS-GMP approach is based on the division of a complex system into smaller systems with unconnected independent DOFs except for a common mechanism attached frame (MAF), as shown in Fig. 1.

Once the model of the full system is split into sub-models, these sub-models are reduced according to the GMP approach such that their behavior can be represented by a reduced unconstrained formulation with a minimal number of DOFs.

Irrespective of the original model formulation (Cartesian coordinates, natural coordinates, . . .) the reduced generalized coordinates q for one sub-model i are:

$$q_i = \begin{bmatrix} x_0 \\ p_0 \\ \theta_i \end{bmatrix}. \tag{1}$$

The reduced sub-model is described by the position x_0 and orientation p_0 of its moving reference frame and the relative mechanism motion with respect to this frame, denoted by a minimal set of coordinates θ_i, as shown in Fig. 1.

Through a nonlinear transformation, the unreduced DOFs of a sub-system x_i can be obtained from the reduced coordinates:

$$x = x_0 + R(p_0)\rho(\theta_i), \tag{2}$$

$$\tag{3}$$

Due to the use of the MAF, a rotation matrix $R(p_0)$ is required, which transforms the coordinates expressed with respect to the local frame to the global frame. In these equations, the nonlinear transformation function $\rho(\theta_i)$ is defined, which relates the unreduced coordinates with respect to the MAF to a minimal set of reduced coordinates θ_i. The derivative of this nonlinear function is defined as[1]:

$$\rho_{,\theta_i} = \Psi^{x\theta_i}, \tag{4}$$

which is a set of projection modes which are also dependent on θ_i. In the GMP formalism, these nonlinear functions are determined by sampling the configuration of the system over a predetermined grid of possible configurations. These configurations are stored and during simulation an Overhauser interpolation is used on this grid, which provides a continuous function between the sampling points up to the first derivative.

2.2 Equations of motion for sub-models

This section briefly reviews the equations of motion for a sub-model. It is important to notice that these equations are the same, irrespective of the original modeling approach. This is a major advantage because it implies that the proposed filtering formalism can be used to generate a KF for any original multibody modeling approach without the necessity to perform any additional derivations of the model to obtain it in a KF-eligible form.

In this section only the final equations are provided, for a full derivation of these equations the reader is referred to [9, 6]. The forces which make up the equations of motion can be split into: inertial forces, internal forces and external forces.

Inertial forces The inertial forces constitute the velocity dependent gyroscopic forces and the acceleration dependent forces.

An additional projection matrix D is required in order to evaluate the inertial forces:

$$D = \begin{bmatrix} R(p_0)^T & 0 \\ 0 & I_{1+m^\theta} \end{bmatrix}. \tag{5}$$

This transformation matrix transforms the sub-system coordinates q_i to the MAF, with m^θ the number of minimal mechanism coordinates. With this projection matrix the sub-system mass-matrix is:

$$M_i = D^T M_c(\theta_i) D. \tag{6}$$

In this equation, $M_c(\theta_i)$ is the reduced mass-matrix with respect to the MAF, which is dependent on the configuration of the system θ_i with respect to the MAF. The reduced mass-matrix is obtained by projecting the unreduced mass-matrix as described by Naets [9], such that the online computations are independent of the unreduced number of DOFs. For the gyroscopic forces, the derivatives of the mass-matrix $M_{i,p_0}, M_{i,\theta_i}$ are required [6]. The gyroscopic forces on DOF j can then be computed as:

$$F^j_{gyr} = \sum_{k=1}^{m^q} \left(M^j_{i,k} \dot{q}^k_i \right) \dot{q}_i - \frac{1}{2} \dot{q}^T_i M_{i,j} \dot{q}_i. \tag{7}$$

Internal forces Even though the current work focuses on rigid multibody systems, internal forces can still be present due to the presence of force elements (eg. springs) in the model. The reduced internal forces $F^{q_i}_{int}$ are obtained by projecting the unreduced (nonlinear) internal forces F^x_{int} onto the reduced DOFs, similar to the process described in [21]:

$$F^{x_0}_{int} = 0, \tag{8}$$

$$F^{p_0}_{int} = 0, \tag{9}$$

$$F^{\theta_i}_{int} = \left(\Psi^{x\theta_i} \right)^T F^x_{int} \left(\rho(\theta_i) \right). \tag{10}$$

[1] the derivative of a matrix A with respect to a variable b is denoted as $A_{,b}$

External forces The external forces are assumed as acting on the unreduced coordinates x. In order to get the effect on the reduced DOFs, they should be projected on the derivative of Eq. (2):

$$F_{ext}^{q_i} = x_{,q_i}^T F_{ext}^x. \tag{11}$$

In order to be able to evaluate the KF properly, also the derivatives of these forces are required:

$$F_{ext,q_i}^{q_i} = x_{,q_i,q_i}^T F_{ext}^x + x_{,q_i}^T F_{ext,x}^x x_{,q_i}. \tag{12}$$

All force contributions, except for the acceleration dependent forces, can be consolidated into a generalized force vector g_i for sub-model i:

$$g_i(q_i, \dot{q}_i, F_{ext}^x) = -F_{int}(q_i) - F_{gyr}(q_i, \dot{q}_i) - x_{,q_i}^T F_{ext}^x, \tag{13}$$

such that the equations of motion for a sub-model can be written as:

$$M_i \ddot{q}_i = g_i(q_i, \dot{q}_i, F_{ext}^x). \tag{14}$$

2.3 First order system equations of motion

In order to evaluate the full system, a back-transformation eliminating the redundant DOFs for the MAFs of the different sub-models has to be performed. This can be obtained by applying a linear projection S onto the coordinates of the n sub-models, such that no additional constraints need to be added to solve the equations of motion [6]:

$$Sq = \begin{bmatrix} q_1 \\ \dots \\ q_n \end{bmatrix}. \tag{15}$$

The full mass-matrix and force vector for the full model is composed from the matrices of the sub-models as:

$$M = S^T \begin{bmatrix} M_1 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & M_n \end{bmatrix} S, \tag{16}$$

$$g(q, \dot{q}, F_{ext}) = S^T \begin{bmatrix} g_1(q_1, \dot{q}_1, F_{ext}^x) \\ \dots \\ g_n(q_n, \dot{q}_n, F_{ext}^x) \end{bmatrix}. \tag{17}$$

With this projection, the equations of motion for all the sub-systems can be combined into the non-redundant equations of motion for the full system:

$$M\ddot{q} = g(q, \dot{q}, F_{ext}). \tag{18}$$

In previous works the equations of motion have always been presented in second-order form. The formulations for state-estimators are however usually derived for first-order systems (exceptions are eg. presented in the work by Hernandez [22]). Therefore, the equations of motion have to be written as:

$$\dot{w} = f(w, F_{ext}), \tag{19}$$

with

$$w = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}. \tag{20}$$

The nonlinear system equation f for the case of these mechanical systems is:

$$f(w, F_{ext}) = \begin{bmatrix} \dot{q} \\ M^{-1} g(q, \dot{q}, F_{ext}) \end{bmatrix}. \tag{21}$$

Furthermore, both the discretization scheme and the KF require the derivative matrix F of this equation:

$$F = \begin{bmatrix} f(w, F_{ext})_{,q} & f(w, F_{ext})_{,\dot{q}} \end{bmatrix}. \tag{22}$$

With these equations, the system can be discretized and the extended Kalman filter can be evaluated. Even though the above derivation was made for the planar case (only one rotation parameter), these results can easily be generalized to the spatial case. In the following section these equations will be exploited to create a coupled state/input-estimator.

3 Augmented discrete extended Kalman filter

In this work the augmented Kalman filter is used, in which the unknown forces a are added as additional states to be estimated. This leads to the augmented state vector w^*:

$$w^* = \begin{bmatrix} w \\ a \end{bmatrix}. \tag{23}$$

The model for the forces is a random walk model [6]:

$$\dot{a} = r_a. \tag{24}$$

In this equation r_a is continuous time noise, which indicates that the rate of change is expected to be a random process. The zeroth order model employed here allows good versatility for different input forces at a minimal computational load.

3.1 Discretization of equations of motion

The discrete extended Kalman filter is most suitable for real-time purposes because it is specifically developed for digital implementation and allows iteration-less integration. In order to be able to apply the DEKF to the continuous time equations of motion of multibody systems, the equations of motion need to be discretized.

With the inclusion of the augmented states for the unknown forces, the continuous time system equations are:

$$\begin{bmatrix} \dot{w} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} f(w, F_{ext} + S_a a) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ r_a \end{bmatrix}. \tag{25}$$

In this equation S_a is a projection matrix which projects the unknown forces to be estimated to the corresponding location in the full force-vector and r_a is the noise-vector which determines the rate of change of the unknown forces. This equation can be summarized as:

$$\dot{w}^* = f^*(w^*, F_{ext}) + r_{w^*}. \tag{26}$$

This augmented function also has a new derivative matrix F^*:

$$F^* = f^*(w^*, F_{ext})_{,w^*}, \tag{27}$$

$$= \begin{bmatrix} F & -M^{-1} x_{,q}^T S_a \\ 0 & 0 \end{bmatrix}. \tag{28}$$

In order to perform the discretization many approaches exist. In this work a nonlinear exponential solver, more specifically the Exponentially Fitted Euler solver, is applied [20, 6]. At a given time t_k this method provides a solution for the states at t_{k+1}:

$$w_{k+1}^* = w_k^* + \int_0^{\Delta t} e^{F_k^* \eta} d\eta \, f^*(w_k^*, F_{ext}) \tag{29}$$

Moreover, also the derivative F_d of this function with respect to the states has to be computed for the Kalman filter [6].

Besides the regular equations of motion, also the discretized behavior of the expected noise on the model has to be considered. In this work, the noise on the continuous time multibody model is assumed to be zero and all noise is assumed on the unknown force, with covariance R_a. The covariance matrix for the discretized time states R_{w^*} is:

$$R_{w^*} \approx F_d^a R_a \left(F_d^a\right)^T. \tag{30}$$

In general R_a is not exactly known and is used as a tuning parameter for the filter in order to get a satisfactory trade-off between the model and the measurements [19, 6].

3.2 A-DEKF algorithm

The augmented discrete extended Kalman filter (A-DEKF) is proposed in this work to provide a coupled estimation of the states and inputs to a mechanical system.

The system equations of motion are complemented by the (nonlinear) measurement equations:

$$y_k = h(w_k^*, F_{ext}) + r_y. \tag{31}$$

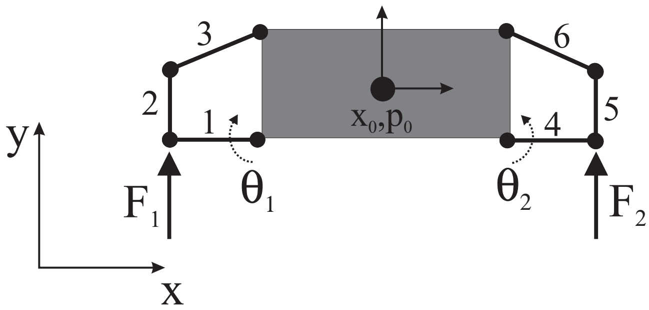


Figure 2: Half-car modeled used for validation.

In this equation y_k contains the sensor measurements obtained by the (nonlinear) measurement equation h with measurement noise r_y with covariance R_y.

For the above described set of equations the DEKF-algorithm for a timestep k becomes [2]:

$$
\begin{aligned}
P_k^- &= F_d P_{k-1}^+ F_d^T + R_{w^*}, & (32)\\
w_k^{*-} &= f_d(w_{k-1}^{*+}, F_{ext}), & (33)\\
K_k &= P_k^- H^T \left(H P_k^- H^T + R_y \right)^{-1}, & (34)\\
w_k^{*+} &= w_k^{*-} + K_k (y_k - h(w_k^{*-}, F_{ext})), & (35)\\
P_k^+ &= (I - K_k H) P_k^-, & (36)
\end{aligned}
$$

with

$$
\begin{aligned}
F_d &= f_d(w_{k-1}^*, F_{ext})_{,w^*}, & (37)\\
H &= h(w_k^*, F_{ext})_{,w^*}. & (38)
\end{aligned}
$$

In this DEKF approach, first an estimate of the states and error-covariance is computed in Eq. (32)-(33). Based on these estimates, the Kalman gain K_k is computed which is used to correct the initial estimates based on the measurements in Eq. (35)-(36). It is interesting to notice that the estimation of the inputs is fully integrated in the regular Kalman filter, the changes are included in the model.

4 Numerical validation

In order to validate the proposed approach, a numerical validation of the A-DEKF with an SS-GMP model is performed [6]. The validation is performed in Matlab and the proposed approach is compared to three different approaches to show consistently superior results. In order to validate the computational efficiency of the A-DEKF with an SS-GMP model, this method is also implemented in FORTRAN.

Firstly the model used is described. Next the different filtering methods are briefly discussed and finally the simulation results are shown.

4.1 Model description

In this work a half-car system, shown in Fig. 2, is used to validate the proposed coupled state/input-estimation approach. The system consists of a car-body and a four-bar suspension on the left and right side. The properties of each body are summarized in Table 1.

The suspension is controlled by a spring at each side. These springs each have a constant stiffness of $40kN/m$ and have a linear force-displacement behavior. No dampers are added in this example.

As external loading, a known gravitational load and vertical wheel forces, which need to be estimated, are applied.

Five measurements: two accelerometer measurements are performed on the horizontal and vertical position of the body. These accelerometers are attached to the car-body and thus provide accelerations in the frame attached to the body:

Furthermore, three gyroscopic measurements are performed: one measurement on the angular velocity of the car body and two measurements for the relative angular velocities of the lower suspension arms. These measurements are chosen because they present a realistic option since they can be performed by low-cost MEMS sensors. For the noise on the

Table 1: Properties of the different bodies of the half-car

	mass $[kg]$	rot. iner. $[kgm^2]$	b $[m]$	h $[m]$
car-body	400	100	0.5	2
beam 1	5	1	0.5	/
beam 2	3	0.7	0.3	/
beam 3	6	1.3	0.5385	/
beam 4	5	1	0.5	/
beam 5	3	0.7	0.3	/
beam 6	6	1.3	0.5385	/

Table 2: Sensor covariances

acc. x_0	$0.8m/s^2$
acc. y_0	$0.8m/s^2$
gyr. p_0	$0.2rad/s$
gyr. θ_1	$0.2rad/s$
gyr. θ_2	$0.2rad/s$

measurements, normal white noise is assumed with realistic values for MEMS sensors. The covariance for the sensors is provided in Table 2.

It is important to notice that these measurements do not lead to an observable system. Over time this will lead to a divergence of the estimated covariance and the Kalman filter will deteriorate [23].

A sampling frequency of $1kHz$ is used in this work for the measurements and the time-integration for the filters is run at the same frequency so there is a measurement for each filtering step.

Reference model The reference model is expressed in Cartesian coordinates for the center of gravity for each body. The constraints on the multibody-system are taken into account through an R-projection approach [24]. The equations of motion are integrated with a generalized α-solver [25] with timestep $1ms$ and spectral radius $\rho_\infty = 0.8$.

SS-GMP model The SS-GMP reduced model consists of 2 sub-models, as shown in Fig. 1. The MAF is attached at the center of gravity of the car body for each sub-model. For the parameterization of the relative mechanism motion θ, the angle between the car-body and the lower-suspension arm is chosen in each model. With these choices each sub-model has four DOFs and the full model has five DOFs. The possible configurations span the range $\theta = [-0.4, 0.63]rad$ with a discretization step of $\Delta\theta = 2mrad$. The storage of the reduced model requires $1.3MB$.

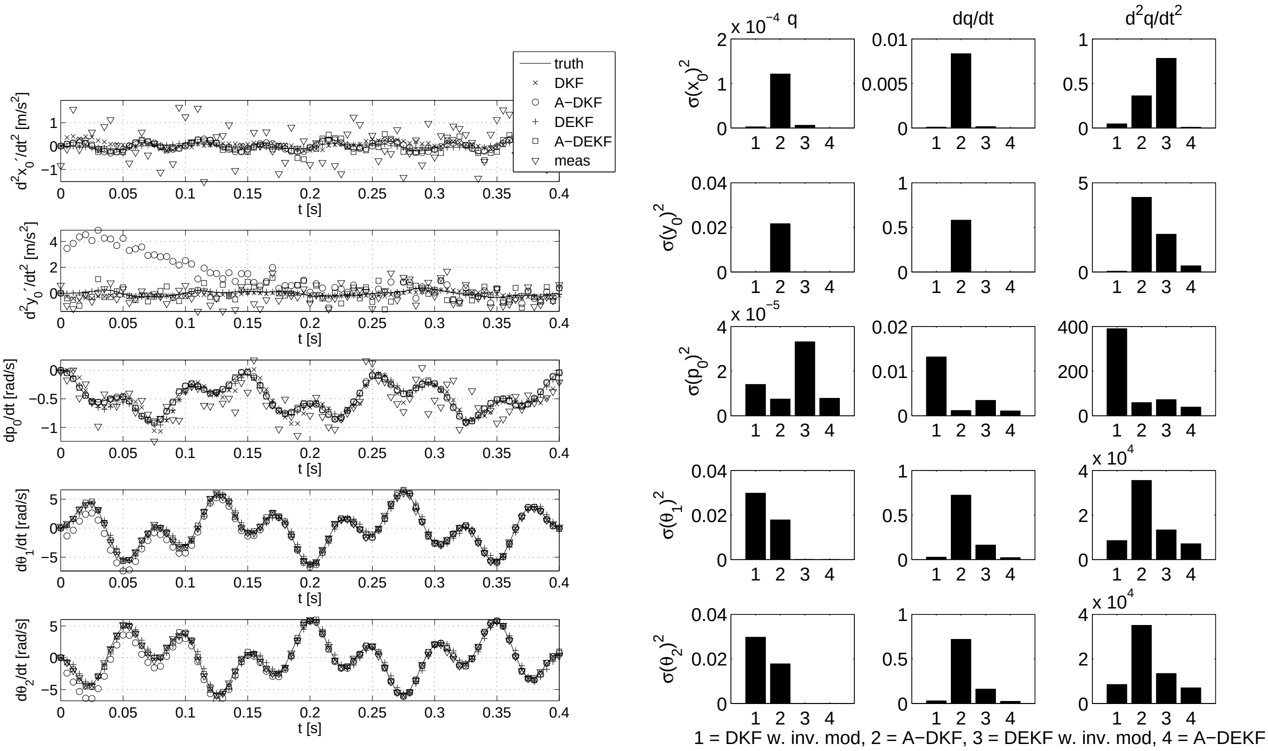
4.2 Filters for comparison

In order to create some frame of reference for the proposed A-DEKF, this method is compared to three other methods:

Separate Kalman filtering with model inversion In this approach each DOF is filtered separately and the force estimates are obtained through a model inversion. This approach is referred to as the *DKF* method in this paper.

A-DKF with linearized model The second estimator which is considered, is the linear augmented discrete Kalman filter (A-DKF). In this approach the system equations are approximated by a linearization of the multibody equations around the initial configuration of the system. This approach is referred to as the *A-DKF* method in this paper.

DEKF with model inversion In this case a discrete extended Kalman filter (DEKF) is applied to the nonlinear SS-GMP model. For this estimator the states are not augmented with the unknown input forces and all forces are assumed constant. This approach is referred to as the *DEKF* method in the the remainder of this chapter.



(a) Measured variables (b) Variance

Figure 3: Tracking of measured variables.

A-DEKF Finally the proposed method in this paper is applied. The augmented discrete extended Kalman filter is applied to the nonlinear SS-GMP model as described in the previous sections with the two unknown external forces as augmented states. The uncertainty concentrated in the rate of change of the unknown input-forces and their covariance after tuning is set at $R_a = 1e6(N/s)^2$. This model is also implemented in FORTRAN in order to verify the real-time capability. This approach is referred to as the *A-DEKF* method in this paper.

4.3 Simulation results

First of all, the tracking behavior for the measurements is compared. Fig. 3 shows the time history of the measurements and their filtered counterparts. All methods provide relatively good results for the tracking of the measured variables. The strength of the Kalman filtering approach is immediately apparent when considering the evolution of the $\ddot{x}_0$-measurement. Fig. 3b shows the A-DKF approach is not able to properly track the behavior of x_0 because this motion is caused by nonlinear couplings in the model which are not present in the linearized model. The DEKF approach also clearly leads to bigger errors than the A-DEKF approach because the unknown input typically leads to biases in the results of the filter.

Furthermore also the behavior of the estimates of the variation on the input forces has to be considered. Fig. 4 shows the course of the two estimated forces. Fig. 4 further enforces the conclusion that the A-DEKF approach is able to deliver accurate results. All three model based approaches (A-DKF, DEKF and A-DEKF) however seem to provide relatively accurate results whereas the DKF method clearly provide inferior performance. This is mainly due to the poor tracking of θ_1 and θ_2 by the DKF approach and these variables are crucial for the spring-forces.

The above results clearly demonstrate that the A-DEKF approach delivers consistently superior results to the other approaches. The FORTRAN simulation of the A-DEKF estimator with the SS-GMP model only takes 0.05 seconds for 0.4 seconds of simulation [2].

[2] All simulations are performed on an Intel Core®2 Duo E6550 2.33GHz processor without exploitation of multi-threading

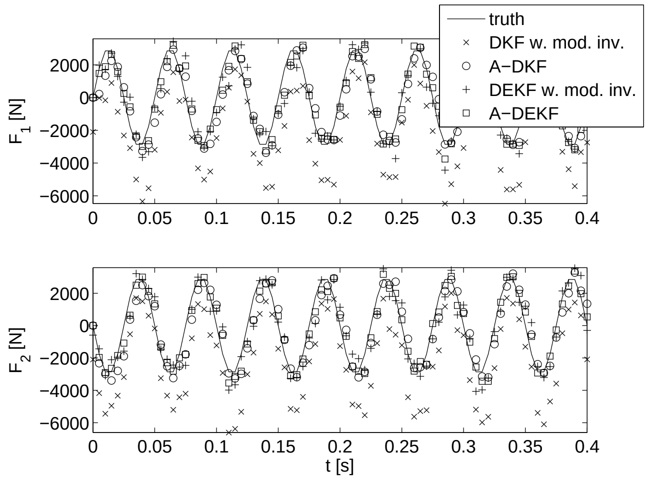


Figure 4: Estimated input forces.

5 Conclusion

In this paper a methodology is proposed to use Sub-System Global Modal Parameterization (SS-GMP) reduced multibody models in an augmented discrete extended Kalman filter (A-DEKF) to generate a general formalism for online coupled state/input estimation for mechanical systems. The use of the SS-GMP approach allows generating real-time capable models from high fidelity multibody models of a mechanical system.

In many mechanical applications it is essential to provide an estimation of the external input forces since these might be very difficult, if not impossible, to determine in advance. In order to allow simultaneous state and input estimation, an augmented Kalman approach is adopted in which the unknown forces are added as additional states to be estimated. A discrete version of the filter is employed because this allows for a more efficient implementation. An additional novelty is the use of an exponential discretization scheme for the nonlinear SS-GMP equations of motion in order to match the model to the filter. Through a numerical validation the accuracy of the proposed A-DEKF filter with SS-GMP model is shown and the FORTRAN implementation of this formalism is able to run faster than real-time on a standard PC.

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