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## A NEW PERFORMANCE INDICATOR FOR IMPACT ANALYSIS WITH APPLICATIONS TO PLANETARY ROVERS

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### ABSTRACT

*An important consideration for the design of planetary rovers and other autonomous mobile robots is the optimization of the design with respect to impact. Impact can occur with the momentary loss of stability as the rover traverses unstructured, extraterrestrial terrain. The analysis of impact often involves the mathematical modelling of the contacting system of bodies to construct the dynamic equations of motion. With these equations, “contact” or “continuous force” models can be used to develop the impact force profile throughout the duration of the impact. While impact force proves to be a good indicator for impact analysis, the contact models used to develop it are highly dependent on the stiffness, damping and other parameters of the system’s physical constitution. Additionally, for the development of these forces, an integration of the equations of motion of the system is required. An alternative to this method that uses the partitioning of the kinetic energy of the system as a performance indicator is proposed here. While a numerical value for impact force is not developed with this method, a qualitative comparison between various impact configurations of the same system can be produced. This spares the computational expense necessary for integration of the dynamic equations, and eliminates the need for information about the constitution of the bodies in the system.*

### INTRODUCTION

The kinetic energy of a system can be partitioned by considering its associations with different subspaces of the system’s motion. Let us consider that the motion of a system can be described with an  $n \times 1$  array of generalized velocities  $\mathbf{v}$ . The vector space spanned by the generalized velocities of the system can be divided into subspaces of particular interest to the analyst. A possible subspace to consider is the subspace spanned by the motion of the system in directions prohibited by the system’s mechanical constraints. Let  $\mathbf{A}$ , an  $m \times n$  matrix be the Jacobian matrix that relates the constrained and generalized velocities. Then these  $m$  constraints can be specified kinematically as  $\mathbf{A}\mathbf{v} = \mathbf{0}$ . This subspace is known as the subspace of constrained motion (SCM). The set of generalized velocities  $\mathbf{v}$  can then be decomposed into components associated with the SCM and its orthogonal complement, the subspace of admissible motion (SAM), as

$$\mathbf{v} = \mathbf{v}_c + \mathbf{v}_a = \mathbf{P}_c \mathbf{v} + \mathbf{P}_a \mathbf{v} \quad (1)$$

where  $\mathbf{P}_c$  and  $\mathbf{P}_a$  are projection matrices onto the SCM and the SAM, respectively. The expression of the projection matrix  $\mathbf{P}_c$  is given by [1]

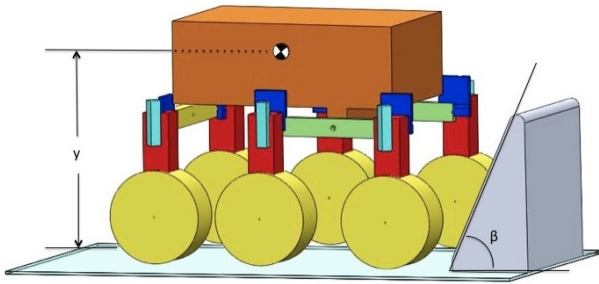
$$\mathbf{P}_c = \mathbf{M}^{-1} \mathbf{A}^T (\mathbf{A} \mathbf{M}^{-1} \mathbf{A}^T)^{-1} \mathbf{A} \quad (2)$$

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Now  $T_c$ , the kinetic energy associated with the SCM can be computed using these projected velocities of the system

$$T_c = \frac{1}{2} \mathbf{v}_c^T \mathbf{M} \mathbf{v}_c \quad (3)$$

where  $\mathbf{M}$  is the  $n \times n$  system mass matrix. In the case of an impact, the instant before the impact occurs, another kinematic specification can be added to the constraint Jacobian, constraining motion for the duration of the impact in the direction normal to the contacting surfaces. Using this updated Jacobian, the value of  $T_c$  that is computed corresponds to the instant just before the impact begins,  $T_c^-$ , and can be used to characterize the maximum value of the normal force during the impact, and the intensity of contact in general.  $T_c^-$ , also termed the “effective kinetic energy”, can be determined for a set of impact situations where the impacts are identical except for one parameter. The one parameter is varied over a range of values, and the effective energy values are recorded and compared. A 3-D model of a rover (Fig. 1) was used to demonstrate the relationship between  $T_c^-$  and the maximum impact force.

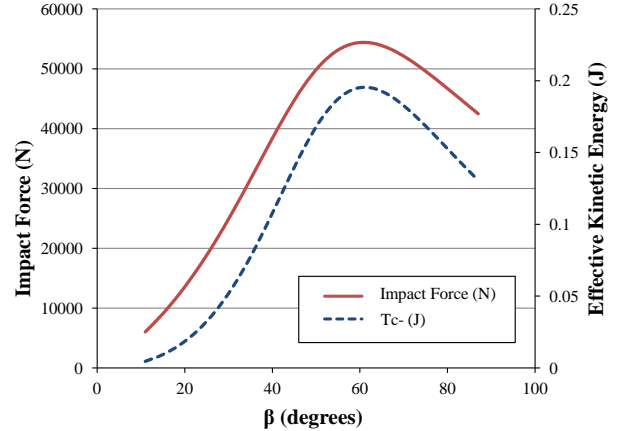


**FIGURE 1.** A 3-D model of a rover undergoing an impact with an obstacle.

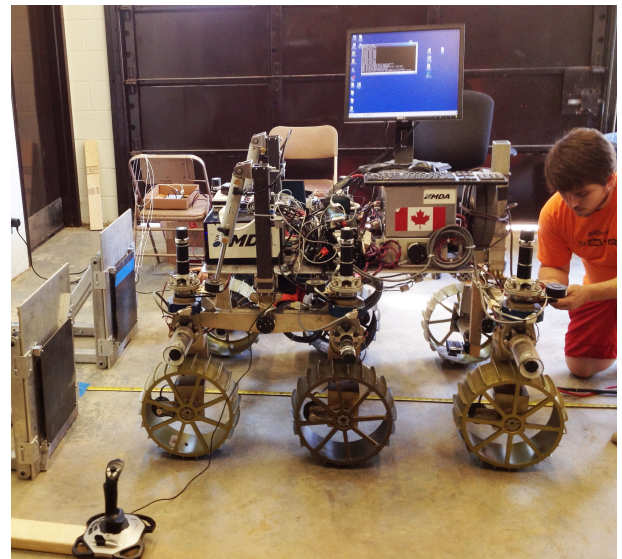
Simulations of the impact of the rover with an obstacle were carried out for different angles of impact  $\beta$ . The effective kinetic energy was evaluated at the instant just before contact was established, and the maximum impact force was determined using the non linear spring-damper model proposed by Hunt and Crossley [2]

Results in Fig. 2 show that the effect of modifying the impact configuration on the impact force  $f_n$  can be captured using  $T_c^-$ . This supports the validity of such an indicator in the estimation of the intensity of impact.

Experimental analysis (Fig. 3) was also carried out with a rover prototype to verify the impact results produced from both performance indicators. The analysis consisted of impacting the prototype on an obstacle constructed so that the angle of its im-



**FIGURE 2.** Maximum impact force and effective energy  $T_c^-$  for impact simulations with a varied angle of impact  $\beta$ . Coefficient of restitution of one.



**FIGURE 3.** Experimental impact set up with a rover prototype.

Impact surface could be incremented from  $0^\circ$  to  $90^\circ$ . Impact sensors mounted on the obstacle recorded the force profile throughout the duration of the experiment.

This effective energy analysis has been used prior to this study in the impact analysis of biomechanical systems [3]. Unlike the rover model however, these systems were modelled using independent coordinates. While the modelling of systems with dependent coordinates produces a more complex constraint Jacobian, it facilitates automation of the analysis where use of independent coordinates can produce difficulty in this area.

## REFERENCES

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