# Reduced order multibody models for the stable co-simulation of multiphysics systems

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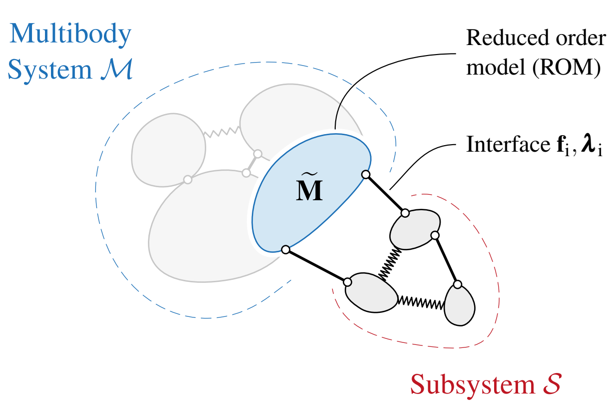
Predictive dynamics simulation is a valuable tool during product development cycle in many industrial applications. The complexity and degree of detail of the computational models of such applications have steadily increased during recent decades, boosted by developments in computing power and simulation software. Nowadays, a significant number of models of complex systems like heavy machinery, road vehicles, or robots consider not only the multibody dynamics aspects of the assembly, but also interactions with other components such as hydraulics and electronics. Often, these possess a dynamic behaviour and time scale that are significantly different from those of the multibody part of the assembly. In these cases, a co-simulation solution can be adopted, in which each sub-system is modelled and integrated separately using a formulation that suits its particular nature. The numerical integration of the sub-systems is synchronized during execution only at discrete communication time instants. Co-simulation simplifies the task of building the computational model of complex engineering applications. However, in some cases it may also lead to the instability of the numerical integration. A variety of techniques have been proposed to alleviate this issue, e.g., integration and communication step-size adjustments [1, 2], and input/output extrapolations [3, 4].

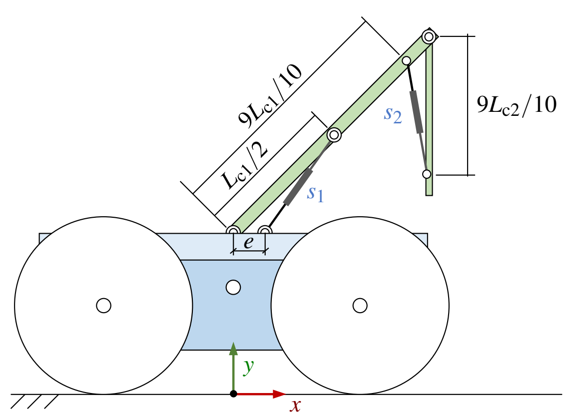
This paper proposes a co-simulation technique that relies on a reduced-order interface model of the multibody subsystem $\mathcal{M}$ (ROM) of a multi-domain, multirate co-simulation setup. This reduced-order model forms the core element in interfacing with another subsystem $\mathcal{S}$ in the co-simulation as conceptually illustrated in Fig. 1a.

The interface between the multibody model and sub-system $\mathcal{S}$ is parametrized by $p$ interface velocities $\mathbf{w}_\mathrm{i}$, which are related to the $n$ multibody system generalized velocities $\mathbf{v}$ through the $p \times n$ interface Jacobian matrix $\mathbf{A}_\mathrm{i}$ as $\mathbf{w}_\mathrm{i} = \mathbf{A}_\mathrm{i}\mathbf{v}$. For a multibody system modelled with non-minimal set of velocities, related by $m$ kinematic constraints $\mathbf{A}\mathbf{v} = \mathbf{0}$, the equivalent ROM consists of the effective mass matrix and force term given by

$$\widetilde{\mathbf{M}} = \left(\mathbf{A}_\mathrm{i}\left(\mathbf{I}-\mathbf{P}_\mathrm{c}\right)\mathbf{M}^{-1}\mathbf{A}_\mathrm{i}^\mathrm{T}\right)^{-1}; \qquad \widetilde{\mathbf{f}} = \widetilde{\mathbf{M}}\left(\mathbf{A}_\mathrm{i}\left(\mathbf{I}-\mathbf{P}_\mathrm{c}\right)\mathbf{M}^{-1}\left(\mathbf{f}-\mathbf{c}\right)+\dot{\mathbf{A}}_\mathrm{i}\mathbf{v}+\mathbf{A}_\mathrm{i}\mathbf{P}_\mathrm{c}\dot{\mathbf{v}}\right) \tag{1}$$

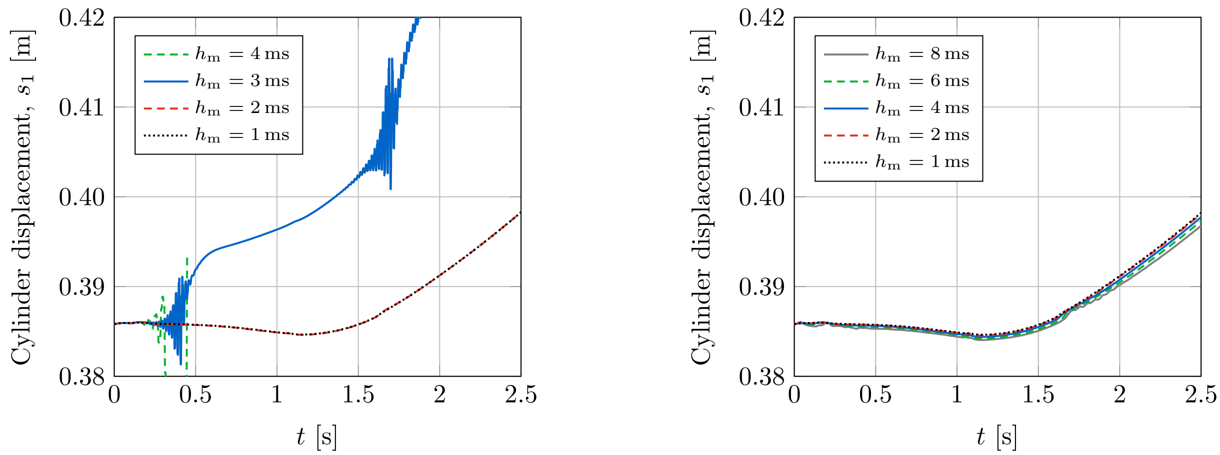



(a) Conceptual illustration




(b) Application: off-road vehicle with hydraulic actuators

Fig. 1: Interfacing a multibody system to an external sub-system via a ROM




(a) Direct ZOH co-simulation

(b) Co-simulation with ROM

Fig. 2: Actuator displacement during a manoeuvre with the vehicle, for different values of the communication step-size $h_m$

where $\mathbf{M}$ is the mass matrix, term $(\mathbf{f}-\mathbf{c})$ contains the forces that act on the system, $\mathbf{I}$ is the $n \times n$ identity matrix, and $\mathbf{P}_c$ is the projector matrix onto the subspace of constrained motion [5]

$$\mathbf{P}_c = \mathbf{M}^{-1}\mathbf{A}^T \left(\mathbf{A}\mathbf{M}^{-1}\mathbf{A}^T\right)^{-1}\mathbf{A} \tag{2}$$

The ROM defined by Eqs. (1) and (2) was used as interface between the mechanical and hydraulic sub-systems in a series of examples, e.g., the off-road vehicle with a hydraulically actuated manipulator shown in Fig. 1b. Results showed that interfacing sub-systems through a ROM can significantly enhance the stability of the simulation This can be a very important feature in the simulation of complex manoeuvres that involve motion discontinuities, such as impacts and unilateral contact. The improved stability behaviour also makes it possible to use larger communication step-sizes for sub-system synchronization, as highlighted in Fig. 2, which resulted in better computational performance.

## References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