Adjoint sensitivity analysis of the index-3 augmented Lagrangian formulation with projections

Daniel Dopico[1], Adrian Sandu[2], Corina Sandu[3].

[1] Laboratorio de Ingeniería Mecánica
Universidade da Coruña
C/ Mendizábal s/n, 15403, Ferrol, Spain
ddopico@udc.es

[2] Department of Computer Science
Virginia Tech
2202, Kraft Dr., 24060, Blacksburg, USA
asandu7@vt.edu

[3] Department of Mechanical Engineering
Virginia Tech
107 Randolph Hall, 24060, Blacksburg, USA
csandu@vt.edu

EXTENDED ABSTRACT

1 Introduction

Design optimization and optimal control of multibody systems usually require the sensitivity analysis of the dynamics of such systems. Dynamic sensitivities, when needed, are often calculated by finite differences but this procedure can be very demanding in terms of time, depending on the number of parameters involved and the accuracy obtained can be very poor in many cases.

Recently, in [1, 2], the forward sensitivity equations for the ALI3-P (Augmented Lagrangian Index-3 formulation with projections) were derived.

The index-3 augmented Lagrangian formulation with velocity and acceleration projections (the ALI3-P formulation) is an efficient and robust method to solve the forward dynamics simulation of general multibody systems, which outperforms the behavior of the aforementioned formulations. It was extensively used for the real-time simulation of different systems with human and hardware in the loop, some of them including complex phenomena like flexibility [3], contact with friction [4, 5] or non-holonomic constraints [6].

In this paper, the adjoint sensitivity equations of the ALI3-P formulation are originally derived and applied to a test case, thus finishing the theoretical sensitivity theory described in [2] for this formulation.

2 Problem statement

Let us consider a multibody system modeled in terms of a set of parameters, $\boldsymbol{\rho} \in \mathbb{R}^p$, with $\mathbf{q}(\boldsymbol{\rho}, t) \in \mathbb{R}^{n_c}$ dependent coordinates related by m holonomic constraints $\boldsymbol{\Phi}(\mathbf{q}, \boldsymbol{\rho}, t) \in \mathbb{R}^m$. The formulation equations of motion (EOM) have the following expressions

$$\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\left(\boldsymbol{\lambda}^{*(i+1)} + \boldsymbol{\alpha}\boldsymbol{\Phi}\right) = \mathbf{Q} \tag{1a}$$

$$\boldsymbol{\lambda}^{*(i+1)} = \boldsymbol{\lambda}^{*(i)} + \boldsymbol{\alpha}\boldsymbol{\Phi}^{(i+1)};\ i > 0 \tag{1b}$$

where $\mathbf{M}(\mathbf{q}, \boldsymbol{\rho}) \in \mathbb{R}^{n_c \times n_c}$ is the mass matrix of the system, $\boldsymbol{\Phi}_{\mathbf{q}}(\mathbf{q}, \boldsymbol{\rho}, t) \in \mathbb{R}^{m \times n_c}$ is the Jacobian matrix of the vector of constraints, $\boldsymbol{\alpha} \in \mathbb{R}^{m \times m}$ is a diagonal matrix containing the penalty factors associated with the constraints, $\mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\rho}, t) \in \mathbb{R}^{n_c}$ is the vector of generalized forces, $i = 0, 1, 2, \ldots$ is the iteration index of the approximate Lagrange multipliers $\boldsymbol{\lambda}^*(\boldsymbol{\rho}, t) \in \mathbb{R}^m$. These converge for $i \to \infty$ to $\boldsymbol{\lambda}$, which are the ones resulting from the solution of the classical index-3 DAE system.

Upon convergence of the equations of motion, the positions $\mathbf{q}$ exactly fulfill the constraint equations $\boldsymbol{\Phi} = \mathbf{0}$, within the convergence tolerance of the algorithm; on the contrary, the satisfaction of $\dot{\boldsymbol{\Phi}} = \mathbf{0}$ and $\ddot{\boldsymbol{\Phi}} = \mathbf{0}$ is not as good and the sets of velocities and accelerations, $\dot{\mathbf{q}}^*$ and $\ddot{\mathbf{q}}^*$ have to be projected onto their corresponding manifolds to obtain their clean counterparts, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ by means of the following expressions:

$$\left(\mathbf{P} + \varsigma\boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\mathbf{q}}\right)\dot{\mathbf{q}} = \mathbf{P}\dot{\mathbf{q}}^* - \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\varsigma\boldsymbol{\alpha}\boldsymbol{\Phi}_t \tag{2}$$

$$\left(\mathbf{P} + \varsigma\boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\mathbf{q}}\right)\ddot{\mathbf{q}} = \mathbf{P}\ddot{\mathbf{q}}^* - \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\varsigma\boldsymbol{\alpha}\left(\dot{\boldsymbol{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} + \dot{\boldsymbol{\Phi}}_t\right) \tag{3}$$

$$\tag{4}$$

Now, let's define a cost function of the dynamic states and parameters,

$$\psi = w\left(\mathbf{q}_F, \dot{\mathbf{q}}_F, \ddot{\mathbf{q}}_F, \boldsymbol{\rho}_F, \boldsymbol{\lambda}_F\right) + \int_{t_0}^{t_F} g\left(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\lambda}, \boldsymbol{\rho}\right) \mathrm{dt}. \tag{5}$$

The problem is to obtain the sensitivity of such a cost function, which is indirectly expressed, in the adjoint sensitivity approach, by means of the following Lagrangian,

$$\begin{aligned}\mathscr{L}(\boldsymbol{\rho}) = {} & w\left(\mathbf{q}_F, \dot{\mathbf{q}}_F, \ddot{\mathbf{q}}_F, \boldsymbol{\rho}_F, \lambda_F\right) + \int_{t_0}^{t_F} g\left(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \lambda, \boldsymbol{\rho}\right) \mathrm{dt} - \int_{t_0}^{t_F} \boldsymbol{\mu}^{\mathrm{T}}\left(\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\left(\boldsymbol{\lambda} + \boldsymbol{\alpha}\boldsymbol{\Phi}\right) - \mathbf{Q}\right) \mathrm{dt} - \int_{t_0}^{t_F} \boldsymbol{\mu}_{\boldsymbol{\Phi}}^{\mathrm{T}}\boldsymbol{\Phi}\mathrm{dt} \\ & - \int_{t_0}^{t_F} \boldsymbol{\mu}_{\dot{\boldsymbol{\Phi}}}^{\mathrm{T}}\left(\left(\mathbf{P} + \varsigma\boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\mathbf{q}}\right)\dot{\mathbf{q}} - \mathbf{P}\dot{\mathbf{q}}^* + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\varsigma\boldsymbol{\alpha}\boldsymbol{\Phi}_t\right) \mathrm{dt} - \int_{t_0}^{t_F} \boldsymbol{\mu}_{\ddot{\boldsymbol{\Phi}}}^{\mathrm{T}}\left(\left(\mathbf{P} + \varsigma\boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\mathbf{q}}\right)\ddot{\mathbf{q}} - \mathbf{P}\ddot{\mathbf{q}}^* - \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\varsigma\boldsymbol{\alpha}\left(\dot{\boldsymbol{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} + \dot{\boldsymbol{\Phi}}_t\right)\right) \mathrm{dt}\end{aligned} \tag{6}$$

and the following identity which holds along any solution of the equations of motion:

$$\nabla_{\boldsymbol{\rho}}\psi = \nabla_{\boldsymbol{\rho}}\mathscr{L} \tag{7}$$

3 Numerical experiment

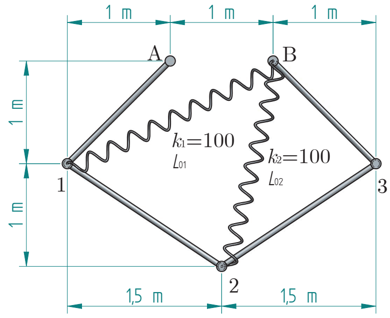

Figure 1: The five-bar mechanism

The test case considered in this work is the five-bar mechanism shown in Figure 1. The sensitivities of this system are well known because they were previously obtained using several different formulations and approaches [7, 8]. The problem posed was the sensitivity analysis of following objective function dependent on the solution of the equations of motion:

$$\psi = \int_{t_0}^{t_F} (\mathbf{r}_2 - \mathbf{r}_{20})^{\mathrm{T}} (\mathbf{r}_2 - \mathbf{r}_{20})\, \mathrm{dt} \tag{8}$$

where $\mathbf{r}_2$ is the global position of the point 2 and $\mathbf{r}_{20}$ is the initial position (at $t = t_0$) of the same point. As parameters to obtain the sensitivities, the natural lengths of the springs were chosen $\boldsymbol{\rho}^{\mathrm{T}} = [L_{01}, L_{02}]$.

The solution offered by the proposed method will be compared with the solutions obtained before and some conclusions will be drawn.

Acknowledgments

The support of the Spanish Ministry of Economy and Competitiveness (MINECO) under project DPI2016-81005-P and the post-doctoral research contract Juan de la Cierva No. JCI-2012-12376 is greatly acknowledged.

References

[1] D. Dopico, F. González, M. Saura, D. G. Vallejo. Forward sensitivity analysis of the index-3 augmented lagrangian formulation with projections. In Proceeding of the 8th ECCOMAS Thematic Conference on Multibody Dynamics. 2017.

[2] D. Dopico, F. González, A. Luaces, M. Saura, D. García-Vallejo. Direct sensitivity analysis of multibody systems with holonomic and nonholonomic constraints via an index-3 augmented lagrangian formulation with projections. Nonlinear Dynamics, 2018. doi:10.1007/s11071-018-4306-y.

[3] J. Cuadrado, R. Gutierrez, M. Naya, P. Morer. A comparison in terms of accuracy and efficiency between a mbs dynamic formulation with stress analysis and a non-linear fea code. International Journal for Numerical Methods in Engineering, 51(9):1033–1052, 2001.

[4] D. Dopico, A. Luaces, M. González, J. Cuadrado. Dealing with multiple contacts in a human-in-the-loop application. Multibody System Dynamics, 25(2):167–183, 2011. doi:10.1007/s11044-010-9230-y.

[5] R. Pastorino, E. Sanjurjo, A. Luaces, M. A. Naya, W. Desmet, J. Cuadrado. Validation of a real-time multibody model for an X-by-wire vehicle prototype through field testing. Journal of Computational and Nonlinear Dynamics, 10(3):031006, 2015. doi:10.1115/1.4028030.

[6] D. Dopico, F. González, J. Cuadrado, J. Kövecses. Determination of holonomic and nonholonomic constraint reactions in an index-3 augmented Lagrangian formulation with velocity and acceleration projections. Journal of Computational and Nonlinear Dynamics, 9(4):041006–041006, 2014. doi:10.1115/1.4027671.

[7] D. Dopico, Y. Zhu, A. Sandu, C. Sandu. Direct and adjoint sensitivity analysis of ordinary differential equation multibody formulations. Journal of Computational and Nonlinear Dynamics, 10(1):1–8, 2014. doi:10.1115/1.4026492.

[8] D. Dopico, A. Sandu, C. Sandu, Y. Zhu. Sensitivity Analysis of Multibody Dynamic Systems Modeled by ODEs and DAEs., chapter 1, 1–32. Multibody Dynamics - Computational Methods and Applications. Springer, 2014. doi:10.1007/978-3-319-07260-9.