Two general index-3 semi-recursive formulations for the dynamics of multibody systems

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EXTENDED ABSTRACT

1 Introduction

A couple of decades ago, the dynamic simulation of complex multibody systems in real-time was an objective difficult to achieve. Not only the fastest formulations available had to be applied, but also powerful and expensive hardware platforms were needed. Nowadays this objective can be easily achieved on low cost PCs for quite complex models like vehicle simulators. However, the field of multibody dynamics incessantly evolves and, if some years ago the objective was to simulate the motion of a mechanical system (sometimes considering complex phenomena like flexible bodies, contact and impact effects, etc.), today other type of problems, more demanding in terms of computational time, need to be solved like, e.g., the design optimization or the optimal control of multibody systems.

Methods developed so far for the dynamic analysis of multibody systems can be grouped into two big families: global and topological. Global methods are characterized by the use of a set of coordinates that perfectly defines the position of each body independently of the rest of the bodies in the system. On the contrary, topological methods make use of joint coordinates (relative) in order to model the mechanism, so that the position of each body is defined with respect to the previous one in the kinematic chain.

MBSLIM [1] is a multibody library with forward kinematics, forward and inverse dynamics and sensitivity analysis of multibody systems, among other capabilities. The library was built on the basis of a natural coordinates formalism plus some angle and distance coordinates (mixed coordinates), i.e., global methods. The extension of the library to accommodate topological formulations is in progress.

With the scope on the extension of MBSLIM some formulations derived in the past [2, 3, 4], all of them based on the articulated inertia method of [5] and similar to those presented in [6], are revisited, generalized and reformulated. The need for generalization of the existing formulations, comes from the fact that the equations involved are not general enough to be integrated in an all-purpose multibody library in natural coordinates with sensitivity analysis capabilities, especially because both set of coordinates need to coexist, the definition of the mechanisms has to be the original one and the library has to be automatic and all the existing models have to work with the new approach.

2 Equations of motion

The equations of motion proposed are the ALI3-P (Index-3 Augmented Lagrangian formulation with projections) semi-recursive equations of motion.

$$\mathbf{M}\ddot{\mathbf{z}}^{*}+\boldsymbol{\Phi}_{\mathbf{z}}^{\mathrm{T}}\left(\boldsymbol{\lambda}^{*(i+1)}+\boldsymbol{\alpha}\boldsymbol{\Phi}\right)=\mathbf{Q}\left(\mathbf{z},\dot{\mathbf{z}}^{*},t\right) \tag{1}$$

$$\boldsymbol{\lambda}^{*(i+1)}=\boldsymbol{\lambda}^{*(i)}+\boldsymbol{\alpha}\boldsymbol{\Phi}^{(i+1)};\, i>0 \tag{2}$$

and where $i=0,1,2,...$, $\boldsymbol{\alpha}$ is a diagonal matrix that contains the penalty factors associated with the constraints, i is the iteration index of the approximate Lagrange multipliers $\boldsymbol{\lambda}^{*}$. These converge for $i\rightarrow\infty$ to $\boldsymbol{\lambda}$, which are the ones resulting from the solution of the original index-3 DAE system. The dynamic terms, $\mathbf{M}$ and $\mathbf{Q}$ have to be calculated recursively depending on the open loop topology of the system (see for example 1). The constraints are the result of imposing the closed loop closure conditions, user constraints (like driving constraints) and maybe especial constraints needed for example for modeling spherical joints with sets of four parameters.

After the solution of the equations of motion, projections of velocities and accelerations are needed to guarantee the stability and accuracy of the solution, obtaining clean velocities and accelerations $\dot{\mathbf{z}}$ and $\ddot{\mathbf{z}}$:

$$\left(\mathbf{P}+\varsigma\boldsymbol{\Phi}_{\mathbf{z}}^{\mathrm{T}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\mathbf{z}}\right)\dot{\mathbf{z}}=\mathbf{P}\dot{\mathbf{z}}^{*}-\varsigma\boldsymbol{\Phi}_{\mathbf{z}}^{\mathrm{T}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{t} \tag{3}$$

$$\left(\mathbf{P}+\varsigma\boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\mathbf{q}}\right)\ddot{\mathbf{z}}=\mathbf{P}\ddot{\mathbf{z}}^{*}-\varsigma\boldsymbol{\Phi}_{\mathbf{z}}^{\mathrm{T}}\boldsymbol{\alpha}\left(\dot{\boldsymbol{\Phi}}_{\mathbf{z}}\dot{\mathbf{z}}+\dot{\boldsymbol{\Phi}}_{t}\right) \tag{4}$$

Matrix $\mathbf{P}$ is the projection matrix.

Moreover, two different methods for the calculation of the dynamic terms, $\mathbf{M}$ and $\mathbf{Q}$, are proposed in the paper, each one of them with advantages and drawbacks.

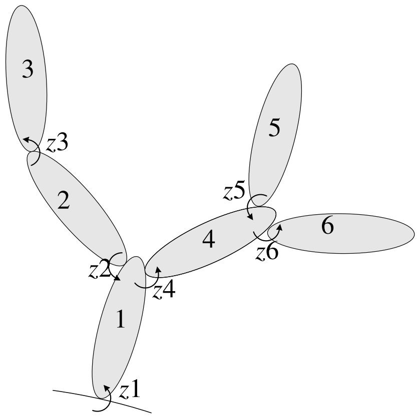

Figure 1: Six body open chain mechanism

3 Numerical experiments

Two important numerical experiments are being solved with the topological formulations implemented. One of them is a closed chain full vecicle model, described in [7] and the other one is the anchor lifting maneuver of ship, which is a novel open chain system with a large amount of bodies involving contact with friction.

Finally a comparison in terms of accuracy and efficiency will be carried out, comparing both topological formulations between them and with the global ALI3-P formulation.

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