Direct Sensitivity Analysis of Spatial Multibody Systems with Friction using Penalty Formulation

Adwait Verulkar[1], Corina Sandu[1], Daniel Dopico[2], Adrian Sandu[3]

[1] Terramechanics, Multibody and Vehicle Systems Laboratory
Dept. of Mechanical Engineering
Virginia Tech
Blacksburg, Virginia 24061
[adwaitverulkar, csandu]@vt.edu

[2] Laboratorio de Ingeniería Mecánica
Dept. of Naval and Industrial Engineering
University of A Coruña
A Coruña, Spain 15001
ddopico@udc.es

[3] Computational Science Laboratory
Dept. of Computer Science
Virginia Tech
Blacksburg, Virginia 24061
sandu@vt.edu

EXTENDED ABSTRACT

1 Introduction

From the onset of the 21st century, the focus of the research in multibody dynamics has shifted to modeling of *real* systems. Such systems are typically characterized by non-linear phenomenon like friction, imperfections in the joints including lubrication and clearances, and systems with discontinuous velocity trajectories, or in other words, hybrid-dynamical systems. Joint friction occurs in all mechanical systems and can possibly have a substantial impact on the dynamics, optimal control, wear, and consequently the operational life of the system. Optimization of such systems is a novel area of research and the methodologies of direct and adjoint sensitivity analysis are the most prominent gradient-based optimization techniques employed for this purpose. Time-based sensitivities are relatively cheaper to compute through direct and adjoint methods as compared to other numerical techniques like finite differences for the same accuracy [1]. Before any of these studies can be conducted, a proper choice of the multibody formulation is necessary. Although Lagrangian formulations are comparatively simple, they contain redundant states (Lagrange multipliers) and require DAE integrators for computation of dynamics and sensitivities. Moreover, Lagrangian formulations are not well suited to handle redundant constraints and singularities. Both of these limitations can be addressed by using the modified Lagrangian formulation, also known as the penalty formulation, which was introduced by Bayo et al. in 1988 [2].

The contribution of this article is in the development of the equations of motion and the methodology of direct sensitivity analysis of multibody systems with joint friction using the penalty formulation. Friction in the system has been represented using the Brown and McPhee friction model [3]. A case study has been conducted on a spatial mechanism and the results of dynamics and sensitivity analysis have been presented. Results of the penalty formulation have also been compared with those obtained through the index-1 Lagrangian formulation.

2 Penalty formulation for multibody systems with joint friction

Bayo et al. [2] modified the Lagrangian formulation based on the Hamiltonian description of dynamics, but instead of appending the constraints $\boldsymbol{\Phi}$ to the formulation, the authors incorporated them in the formulation itself using a penalty matrix $\boldsymbol{\alpha}$. This approach leads to a system of n ordinary differential equations (ODEs) as opposed to the $n+m$ differential algebraic equations (DAEs) of the classical Lagrangian formulations. The equations of motion can be expressed in a compact ODE form as follows

$$\bar{\mathbf{M}}\,\ddot{\mathbf{q}} = \bar{\mathbf{Q}} \tag{1}$$

where

$$\bar{\mathbf{M}} = \mathbf{M} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}} \boldsymbol{\alpha}\, \boldsymbol{\Phi}_{\mathbf{q}} \tag{2}$$

$$\bar{\mathbf{Q}} = \mathbf{Q} + \mathbf{Q}^{Af*} - \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}} \boldsymbol{\alpha} \left(\dot{\boldsymbol{\Phi}}_{\mathbf{q}} \dot{\mathbf{q}} + \dot{\boldsymbol{\Phi}}_t + 2\,\xi\,\omega \dot{\boldsymbol{\Phi}} + \omega^2\, \boldsymbol{\Phi} \right) \tag{3}$$

In penalty formulation, the Lagrange multipliers are approximated with the following term

$$\boldsymbol{\lambda}^* = \boldsymbol{\alpha} \left(\ddot{\boldsymbol{\Phi}} + 2\,\xi\,\omega \dot{\boldsymbol{\Phi}} + \omega^2\, \boldsymbol{\Phi} \right) \tag{4}$$

Since, the exact Lagrange multipliers do not exist in penalty formulation, the friction force vector $\mathbf{Q}^{Af*}$ is calculated based on the approximate Lagrange multipliers given in Equation (4). The term that depends on the Lagrange multipliers in $\mathbf{Q}^{Af*}$ is actually the magnitude of the joint reaction force F_n^*. Based on Haug (2018) [4], the equation for the joint reaction force in the joint-fixed reference frame for a body i using approximate Lagrange multipliers can be written as

$$F_n^* = \left| \mathbf{F}_i^{\prime\prime k} \right| = \left| -\mathbf{C}_i^{k\mathrm{T}}\, \mathbf{A}_i^{\mathrm{T}}\, \boldsymbol{\Phi}_{\mathbf{r}_i}^{k\mathrm{T}} \boldsymbol{\lambda}^{*k} \right| \tag{5}$$

In penalty formulation, neither of the constraint equations nor their derivatives are exactly satisfied, however, reasonable accuracy can be achieved through a right choice of penalty factors, natural frequency and damping ratio. In this analysis, we have considered $\alpha = 1000$, $\xi = 1$ and $\omega = 10$ rad/s.

3 Direct sensitivity analysis for multibody systems with joint friction

The direct differentiation method for the sensitivity analysis using the penalty formulation was first presented by Pagalday (1997) [5]. To obtain the expression for the model sensitivities, Equation (1) is differentiated with respect to the model parameters.

$$\frac{\mathrm{d}\bar{\mathbf{M}}}{\mathrm{d}\rho_k}\ddot{\mathbf{q}}+\bar{\mathbf{M}}\frac{\mathrm{d}\ddot{\mathbf{q}}}{\mathrm{d}\rho_k}=\frac{\mathrm{d}\bar{\mathbf{Q}}}{\mathrm{d}\rho_k}, \qquad k=1,2,\ldots,p \tag{6}$$

The derivatives can be expanded and rearranged into p Tangent Linear Models (TLMs) as follows

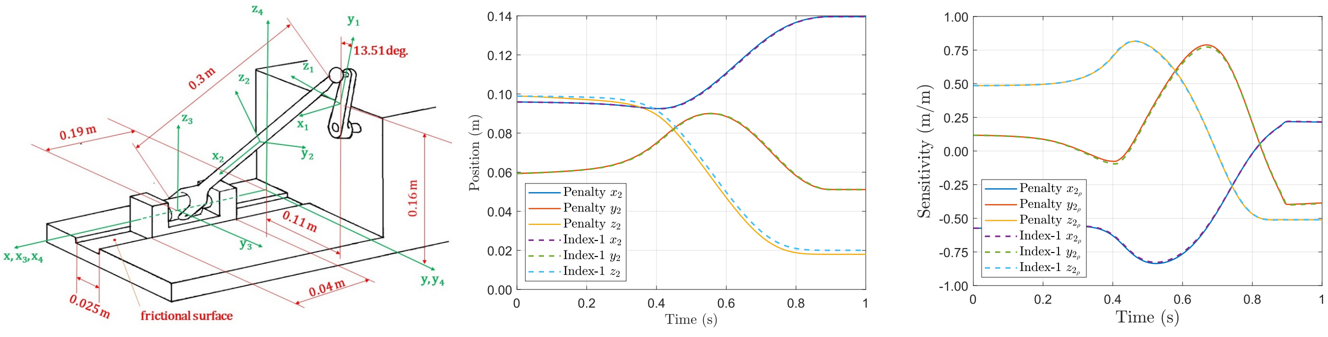
$$\bar{\mathbf{M}}\ddot{\mathbf{q}}_{\boldsymbol{\rho}}+\bar{\mathbf{C}}\dot{\mathbf{q}}_{\boldsymbol{\rho}}+(\bar{\mathbf{K}}+\bar{\mathbf{M}}_{\mathbf{q}}\ddot{\mathbf{q}})\mathbf{q}_{\boldsymbol{\rho}}+\mathbf{L}^{Af*}\boldsymbol{\lambda}^*_{\boldsymbol{\rho}}=\bar{\mathbf{Q}}_{\boldsymbol{\rho}}-\bar{\mathbf{M}}_{\boldsymbol{\rho}}\ddot{\mathbf{q}} \tag{7}$$

The term $\boldsymbol{\lambda}^*_{\boldsymbol{\rho}}$ represents the sensitivities of the approximate Lagrange multipliers with respect to the design parameters $\boldsymbol{\rho}$. These can be expressed in terms of the sensitivities of the generalized coordinates and their derivatives. This substitution yields the final form of the Tangent Linear Model as shown below

$$\begin{aligned}(\bar{\mathbf{M}}+\boldsymbol{\alpha}\mathbf{L}^{Af*}\boldsymbol{\Phi}_{\mathbf{q}})\ddot{\mathbf{q}}_{\boldsymbol{\rho}}+(\bar{\mathbf{C}}+\boldsymbol{\alpha}\mathbf{L}^{Af*}(\dot{\boldsymbol{\Phi}}_{\dot{\mathbf{q}}}+2\xi\omega\boldsymbol{\Phi}_{\mathbf{q}}))\dot{\mathbf{q}}_{\boldsymbol{\rho}}+(\bar{\mathbf{K}}+\bar{\mathbf{M}}_{\mathbf{q}}\ddot{\mathbf{q}}+\boldsymbol{\alpha}\mathbf{L}^{Af*}(\ddot{\boldsymbol{\Phi}}_{\mathbf{q}}+2\xi\omega\dot{\boldsymbol{\Phi}}_{\mathbf{q}}+\omega^2\boldsymbol{\Phi}_{\mathbf{q}}))\mathbf{q}_{\boldsymbol{\rho}}\\=\bar{\mathbf{Q}}_{\boldsymbol{\rho}}-\bar{\mathbf{M}}_{\boldsymbol{\rho}}\ddot{\mathbf{q}}-\boldsymbol{\alpha}\mathbf{L}^{Af*}(\ddot{\boldsymbol{\Phi}}_{\boldsymbol{\rho}}+2\xi\omega\dot{\boldsymbol{\Phi}}_{\boldsymbol{\rho}}+\omega^2\boldsymbol{\Phi}_{\boldsymbol{\rho}})\end{aligned} \tag{8}$$

4 Results and conclusion

A case study was conducted on a spatial slider crank mechanism with joint friction to validate the proposed methodology. This mechanism was adapted from Haug (1989) [6] and the schematic of this system is shown in Figure 1(a). For comparison, the dynamics and sensitivities of the connecting rod with respect to the crank length have been plotted in Figures 1(b) and 1(c) respectively using the index-1 Lagrangian and penalty formulations. The computation using the penalty formulation was found to be 26% faster than the index-1 formulation with the maximum RMSE error in sensitivities of 0.5% for a simulation time of 1 second.



(a) (b) (c)

Figure 1: (a) Mechanism schematic. (b) Connecting rod position history. (c) Sensitivity of rod position to crank length.

Acknowledgments

The work has been supported in part by the Robert E. Hord Jr. Professorship and by the Terramechanics, Multibody, and Vehicle Systems Laboratory at Virginia Tech.

References

[1] Y. Zhu, C. Sandu, D. Dopico, and A. Sandu, "Benchmarking of adjoint sensitivity-based optimization techniques using a vehicle ride case study," *Mechanics Based Design of Structures and Machines*, vol. 46, no. 2, pp. 254–266, 2018.

[2] E. Bayo, J. García de Jalón, and M. A. Serna, "A modified lagrangian formulation for the dynamic analysis of constrained mechanical systems," *Computer Methods in Applied Mechanics and Engineering*, vol. 71, no. 2, pp. 183–195, 1988.

[3] P. Brown and J. McPhee, "A Continuous Velocity-Based Friction Model for Dynamics and Control with Physically Meaningful Parameters," *Journal of Computational and Nonlinear Dynamics*, vol. 11, no. 5, pp. 1–6, 2016.

[4] E. J. Haug, "Simulation of spatial multibody systems with friction," *Mechanics Based Design of Structures and Machines*, vol. 46, no. 3, pp. 347–375, 2018. [Online]. Available: https://doi.org/10.1080/15397734.2017.1377086

[5] J. M. Pagalday and A. Avello, "Optimization of multibody dynamics using object oriented programming and a mixed numerical-symbolic penalty formulation," *Mechanism and Machine Theory*, vol. 32, no. 2, pp. 161–174, Feb 1997. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0094114X96000377

[6] E. J. Haug, *Computer aided kinematics and dynamics of mechanical systems, volume 1: Basic methods*, Massachusetts, 1989.