Optimization of a Three Wheeled Tilting Vehicle.

Daniel Dopico Dopico, Álvaro López Varela, Alberto Luaces Fernández.

Laboratorio de Ingeniería Mecánica
Universidade da Coruña
C/ Mendizábal s/n, 15403, Ferrol, Spain
[ddopico,alvaro.lopez1,aluaces]@udc.es

EXTENDED ABSTRACT

1 Introduction

Three wheeled tilting vehicles are an alternative to common bicycles. The dynamics ot such a vehicle can be made similar to the bicycle dynamics, nevertheless the system offers wider configuration and desing possibilities with a larger variety of behaviors. The simpler and safer strategy is to design the vehicle to mimic the “equivalent” bicycle dynamics, but even in this case the engineer must face additional design problems compared to a common bicycle design.

The steering optimal design of the tadpole tilting three wheeled vehicle multibody model shown in Figure 1 is not an easy task. The steering system should satisfy Ackerman’s steering condition, not only for null roll angles (the typical design for a car steering) but also for any combination of roll and steering angles. Moreover, the relation between the handlebar rotation and the wheels angles should be adjusted. In case we wish to mimic a standard bicicle behavior, this relation mus be approximately equivalent to the single handlebar-wheel mount of a common bicycle.

In this work, the optimization of the mentioned vehicle, paying especial attention to the steering system is addressed. Several optimization problems are solved: first the kinematic design optimization of the steering; second the dynamic optimization of the steering, equivalent to the kinematic optimization but solved under dynamic conditions, making possible to desing the system to real-drive situations; third, the optimal design of the system, which can be used to program some maneuvers for the dynamical design optimization. All the optimizations performed are gradient-based, they are solved under the same general framework and rely on the multibody sensitivity equations using two approaches: direct sensitivity for optimal design and adjoint sensitivity for optimal control.

2 Kinematic problem statement

Let us consider a multibody system modeled with $\mathbf{q} \in \mathbb{R}^n$ dependent coordinates related by $\mathbf{\Phi} \in \mathbb{R}^m$ holonomic constraints. Only d coordinates out of the full set of n are independent and they can be chosen as degrees of freedom of the system, $\mathbf{z} \in \mathbb{R}^d$. The kinematic equations at position level can be represented as:

$$\begin{bmatrix} \mathbf{\Phi}_{\mathbf{q}}^{\{i\}} \\ \mathbf{B} \end{bmatrix} \Delta\mathbf{q}^{\{i+1\}} = \begin{bmatrix} -\mathbf{\Phi}^{\{i\}} \\ \mathbf{0} \end{bmatrix}; \quad i = 0, 1, 2, ..., \tag{1a}$$

$$\Delta\mathbf{q}^{\{i+1\}} = \mathbf{q}^{\{i+1\}} - \mathbf{q}^{\{i\}} \tag{1b}$$

$$\mathbf{\Phi}^{\{i\}} = \mathbf{\Phi}\left(t, \mathbf{q}^{\{i\}}, \boldsymbol{\rho}\right) \tag{1c}$$

Observe that the constraint equations depend on some design parameters, normally local coordinates of points or vectors defining the model. These parameters are of interest for the optimization to accomplish.

3 Dynamic problem statement

Let us consider a multibody system modeled with $\mathbf{q} \in \mathbb{R}^n$ dependent natural coordinates related by $\mathbf{\Phi} \in \mathbb{R}^m$ holonomic constraints. Consider the dynamics of the system dependent on some parameters $\boldsymbol{\rho} \in \mathbb{R}^p$, being some of them design parameters, i.e., local coordinates of points, parameters related to masses or forces; while some others can be control function parameters affecting forces or rheonomic constraints. All of them, design parameters and optimal controls, are considered under the same framework.

The equations of motion for the system can be represented as:

$$\mathbf{M}\ddot{\mathbf{q}}^* + \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\left(\boldsymbol{\lambda}^{*(i+1)} + \boldsymbol{\alpha}\mathbf{\Phi}\right) = \mathbf{Q} \tag{2}$$

$$\boldsymbol{\lambda}^{*(i+1)} = \boldsymbol{\lambda}^{*(i)} + \boldsymbol{\alpha}\mathbf{\Phi}; \, i > 0 \tag{3}$$

where $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the mass matrix of the system, $\mathbf{\Phi}_{\mathbf{q}} \in \mathbb{R}^{m\times n}$ is the jacobian matrix of the constraints, $\mathbf{Q} \in \mathbb{R}^n$ is the generalized forces vector and $\boldsymbol{\lambda}^* \in \mathbb{R}^m$ the Lagrange multipliers.

In this formulation, the fulfillment of the constraints in velocities and accelerations is imposed with velocity and acceleration projections:

$$\left(\bar{\mathbf{P}}+\varsigma\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\mathbf{\Phi}_{\mathbf{q}}\right)\dot{\mathbf{q}}=\bar{\mathbf{P}}\dot{\mathbf{q}}^{*}-\varsigma\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\mathbf{\Phi}_{t} \tag{4}$$

$$\left(\bar{\mathbf{P}}+\varsigma\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\mathbf{\Phi}_{\mathbf{q}}\right)\ddot{\mathbf{q}}=\bar{\mathbf{P}}\ddot{\mathbf{q}}^{*}-\varsigma\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\left(\dot{\mathbf{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}}+\dot{\mathbf{\Phi}}_{t}\right) \tag{5}$$

Where $\bar{\mathbf{P}}$ is a symmetric projection matrix, and the superscript * indicates that the correspondent term is an unprojected magnitude.

4 Optimization and optimal control problem statement

Let us consider a set of objective functions, $\boldsymbol{\psi}\in\mathbb{R}^{o}$, expressed as integrals in time:

$$\boldsymbol{\psi}=\int_{t_0}^{t_F}\mathbf{g}\left(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}},\boldsymbol{\lambda},\boldsymbol{\rho}\right)\mathrm{d}t. \tag{6}$$

The sensitivity analysis of the objective functions with respect to the set of parameters $\boldsymbol{\rho}\in\mathbb{R}^{p}$ can be computed by means of direct sensitivity or adjoint sensitivity methods and using the kinematic or the dynamic equations presented before [1, 2].

5 Numerical experiments

The case study for optimal design and optimal control is the tilting three wheeled vehicle shown in Figure 1. The optimal design can be accomplished by means of a kinematic analysis in positions or by means of a dynamic analysis in order to better optimize for the service conditions of the vehicle.

The objective functions considered enforce the satisfaction of Ackerman's steering principle and the relation between the handlebar rotation and the effective steering angle. For the dynamic simulation, the degrees of freedom of the vehicle are predeterminated and the optimization is carried out over this prescribed motion, but for the dynamic simulation, an optimal control function will be added to force the vehicle to fit the desired trajectory and speed, controlling the handlebar and pedals.

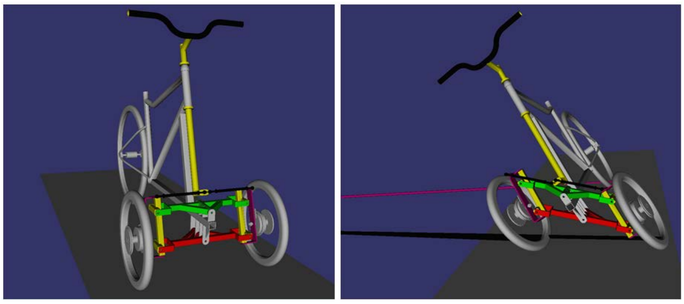

Figure 1: Three wheeled tilting vehicle.

6 Conclusions

The present work proves that the approach proposed is a valid approach to improve the design of mechanical systems using kinematics or dynamics simulations. Moreover, the optimal control is also considered and both types of problems can be solved together under the same framework.

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References

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