Sensitivity analysis of semi-recursive augmented Lagrangian formulations with projections

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1 Introduction

Lately, the multibody community has payed special attention to the sensitivity analysis of multibody dynamics (MBD). There are numerous works related to the study of the sensitivities of different dynamic multibody formulations applying the direct differentiation method (DDM) or the adjoint variable method (AVM), but those works are generally posed in terms of reference point coordinates or natural coordinates. Deep and thorough studies on the sensitivity analysis of relative coordinate multibody models are seldom, and are usually addressed by means of automatic differentiation or numerical differentiation.
In the present work, the sensitivity analysis of the semi-recursive index-3 Augmented Lagrangian Formulation with velocity and acceleration projections described in [1] is unfolded applying the DDM and the AVM, using the so called RTdyn0 and RTdyn1 procedures for the generation of the Equations of Motion (EoM). In this case, the robustness of the dynamics are translated to the sensitivities, supporting singular mass matrices, redundant constraints and singular configurations. The sensitivity analyses presented have been implemented as general sensitivity formulations in the MBSLIM multibody library [2].

2 Forward sensitivity

Let us consider an objective function $\boldsymbol{\psi} \in \mathbb{R}^o$ expressed in terms of a relative coordinates vector $\mathbf{z} \in \mathbb{R}^n$ and its time derivatives $\dot{\mathbf{z}}, \ddot{\mathbf{z}} \in \mathbb{R}^n$, Lagrange multipliers $\boldsymbol{\lambda}^* \in \mathbb{R}^{nc}$, and a set of parameters $\boldsymbol{\rho} \in \mathbb{R}^p$. The gradient of this objective function can be expressed as:

$$\boldsymbol{\psi}' = \nabla \boldsymbol{\psi}^{\mathrm{T}} = \left(\mathbf{w}_{\hat{\mathbf{z}}}\mathbf{z}' + \mathbf{w}_{\hat{\dot{\mathbf{z}}}}\dot{\mathbf{z}}' + \mathbf{w}_{\hat{\ddot{\mathbf{z}}}}\ddot{\mathbf{z}}' + \mathbf{w}_{\boldsymbol{\lambda}^*}\boldsymbol{\lambda}^{*\prime} + \mathbf{w}_{\hat{\boldsymbol{\rho}}}\right)_F + \int_{t_F}^{t_0} \left(\mathbf{g}_{\hat{\mathbf{z}}}\mathbf{z}' + \mathbf{g}_{\hat{\dot{\mathbf{z}}}}\dot{\mathbf{z}}' + \mathbf{g}_{\hat{\ddot{\mathbf{z}}}}\ddot{\mathbf{z}}' + \mathbf{g}_{\boldsymbol{\lambda}^*}\boldsymbol{\lambda}^{*\prime} + \mathbf{g}_{\hat{\boldsymbol{\rho}}}\right) \mathrm{d}t \quad (1)$$

In which $(.)_{\hat{\mathbf{z}}}$, $(.)_{\hat{\dot{\mathbf{z}}}}$, $(.)_{\hat{\ddot{\mathbf{z}}}}$ and $(.)_{\hat{\boldsymbol{\rho}}}$ gather partial derivatives with respect to $\mathbf{z}$, $\dot{\mathbf{z}}$, $\ddot{\mathbf{z}}$ and $\boldsymbol{\rho}$ respectively considering intermediate implicit dependencies, and $(.)'$ denotes state derivatives. Behold that derivatives of $\mathbf{g} \in \mathbb{R}^o$ and $\mathbf{w} \in \mathbb{R}^o$ are known while the sensitivities $\mathbf{z}'$, $\dot{\mathbf{z}}'$, $\ddot{\mathbf{z}}'$ and $\boldsymbol{\lambda}^{*\prime}$ are unknown magnitudes.
Applying the direct differentiation method to the ALI3-P EoM, these unknown magnitudes can be obtained following the same scheme introduced in [3]. This leads to p index-3 differential-algebraic systems of equations (DAE) plus $2p$ systems of algebraic equations for the sensitivity of the velocity and acceleration projections.

3 Adjoint sensitivity

The DDM yields a set of systems of equations whose quantity is proportional to the number of parameters and whose main variables are the sensitivities of the states and Lagrange multipliers. In order to circumvent the computational inefficiency of the DDM for systems subjected to numerous parameters, the AVM is resorted to.
The AVM is a well known sensitivity technique which allows to reduce the number of systems of equations required to obtain the gradient of any objective function by means of a reformulation of the sensitivity problem. The method is based on the generation of a Lagrangian function including the set of equations solved in the dynamics and a set of new variables, namely the adjoint variables.

$$\begin{aligned}\mathscr{L} = \boldsymbol{\psi} - \int_{t_0}^{t_F} \boldsymbol{\mu}_2^{\mathrm{T}} \left(\mathbf{M}\dot{\boldsymbol{\nu}}^* + \boldsymbol{\Phi}_{\hat{\mathbf{z}}}^{\mathrm{T}}\left(\boldsymbol{\lambda}^* + \boldsymbol{\alpha}\boldsymbol{\Phi}\right) - \mathbf{Q}\right) \mathrm{d}t - \int_{t_0}^{t_F} \boldsymbol{\mu}_1^{\mathrm{T}} \left(\dot{\mathbf{z}}^* - \boldsymbol{\nu}^*\right) \mathrm{d}t - \int_{t_0}^{t_F} \boldsymbol{\mu}_{\boldsymbol{\Phi}}^{\mathrm{T}} \boldsymbol{\Phi} \mathrm{d}t \\ - \int_{t_0}^{t_F} \boldsymbol{\mu}_{\dot{\boldsymbol{\Phi}}}^{\mathrm{T}} \left(\left[\bar{\mathbf{P}} + \varsigma \boldsymbol{\Phi}_{\hat{\mathbf{z}}}^{\mathrm{T}} \boldsymbol{\alpha} \boldsymbol{\Phi}_{\hat{\mathbf{z}}}\right] \dot{\mathbf{z}} - \bar{\mathbf{P}}\boldsymbol{\nu}^* + \boldsymbol{\Phi}_{\hat{\mathbf{z}}}^{\mathrm{T}} \varsigma \boldsymbol{\alpha} \boldsymbol{\Phi}_t\right) \mathrm{d}t \quad (2) \\ - \int_{t_0}^{t_F} \boldsymbol{\mu}_{\ddot{\boldsymbol{\Phi}}}^{\mathrm{T}} \left(\left[\bar{\mathbf{P}} + \varsigma \boldsymbol{\Phi}_{\hat{\mathbf{z}}}^{\mathrm{T}} \boldsymbol{\alpha} \boldsymbol{\Phi}_{\hat{\mathbf{z}}}\right] \ddot{\mathbf{z}} - \bar{\mathbf{P}}\dot{\boldsymbol{\nu}}^* + \boldsymbol{\Phi}_{\hat{\mathbf{z}}} \varsigma \boldsymbol{\alpha} \left(\dot{\boldsymbol{\Phi}}_{\hat{\mathbf{z}}} \dot{\mathbf{z}} + \dot{\boldsymbol{\Phi}}_t\right)\right) \mathrm{d}t + \boldsymbol{\eta}^{\mathrm{T}} \dot{\boldsymbol{\Phi}} \left(t_F, \mathbf{q}_F, \boldsymbol{\nu}_F^*, \boldsymbol{\rho}\right)\end{aligned}$$

In the case of ALI3-P formulations, this problem has been addressed in [4], but as the authors state, it has been proved with natural coordinate models, this is, with constant mass and projection matrices. In this work, the formalism presented in [4] is extended to semi-recursive formulations.

4 Numerical experiments

Both DMM and AVM have been tested against other formulations in natural coordinates and against numerical differentiation. A numerical experiment has been considered in this regard: the five-bar benchmark mechanism described in [3, 4]. The objective function considered is:

$$\Psi = \int_{t_0}^{t_F} (\mathbf{r}_2 - \mathbf{r}_{20})^{\mathrm{T}} (\mathbf{r}_2 - \mathbf{r}_{20}) \, \mathrm{d}t \tag{3}$$

in which $\mathbf{r}_2$ and $\mathbf{r}_{20}$ represent the point 2 of the mechanism at any time and at the initial time respectively. In order to check the assembly processes, in addition to the natural lengths of the springs the mass, center of mass and length of the first bar of the mechanism are included in the set of parameters of the system. Table 1 demonstrates the accuracy of the method for any of the parameters considered.

	DDM		AVM		
	RTdyn0	RTdyn1	RTdyn0	RTdyn1	Reference
$\left(\boldsymbol{\psi}^1\right)'_{L_{s1}}$	-4.228	-4.228	-4.228	-4.228	-4.228
$\left(\boldsymbol{\psi}^1\right)'_{L_{s2}}$	3.212	3.212	3.212	3.212	3.212
$\left(\boldsymbol{\psi}^1\right)'_{m_{A1}}$	0.3186	0.3186	0.3186	0.3186	0.3186
$\left(\boldsymbol{\psi}^1\right)'_{x^G_{A1}}$	0.4423	0.4423	0.4423	0.4423	0.4423
$\left(\boldsymbol{\psi}^1\right)'_{L_{A1}}$	3.360	3.360	3.360	3.360	3.360

Table 1: Gradient of the cost function of the five-bar mechanism.

5 Conclusions

In this work, the sensitivity analysis of the semi-recursive ALI3-P formulation using two different schemes of generation of the EoM (RTdyn0 and RTdyn1) and two different methods (DDM and AVM) has been presented and tested in a numerical experiment. The authors would like to point out that the main novelty of this work is included in the calculation and assembly of the derivatives of semi-recursive mass matrices, forces and constraints in a general fashion for any type of joint, constraint and topology.

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References

[1] D. Dopico, Á. López, A. Luaces. Augmented lagrangian index-3 semi-recursive formulations with projections. Multibody System Dynamics, 2020. doi:https://doi.org/10.1007/s11044-020-09771-9.

[2] D. Dopico, A. Luaces, U. Lugrís, M. Saura, F. González, E. Sanjurjo, R. Pastorino. Mbslim: Multibody systems en laboratorio de ingeniería mecánica, 2009-2016.

[3] D. Dopico, F. González, A. Luaces, M. Saura, D. García-Vallejo. Direct sensitivity analysis of multibody systems with holonomic and nonholonomic constraints via an index-3 augmented lagrangian formulation with projections. Nonlinear Dynamics, 2018. doi:10.1007/s11071-018-4306-y.

[4] D. Dopico, A. Sandu, C. Sandu. Adjoint sensitivity index-3 augmented lagrangian formulationwith projections. Mechanics Based Design of Structures and Machines, 2021. doi: 10.1080/15397734.2021.1890614.