

# Kalman filter based on multibody models with unknown statistical properties

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## EXTENDED ABSTRACT

### 1 Introduction

During last years, the importance of connecting real and virtual world has increased. The use of digital twins is extended into several industry fields. For this purpose, it is usually required to synchronize a virtual model with the real system through the use of sensors. However, in many applications, there are variables that cannot be measured through physical sensors. As an alternative, those variables can be estimated via a Kalman filter.

The Kalman filter is based on a model of the system and a reduced set of sensors. The information acquired through the sensors is employed by the filter to correct the possible drift between the model and reality. The Kalman filter was initially developed for linear systems. However, different approaches have been derived so that it can be applied to non-linear systems, such as multibody models [1]. One of the most interesting approaches when using multibody models is the error extended Kalman filter with force estimation (errorEKF\_FE). It is an indirect Kalman filter which estimates the errors in the evaluation of the states, together with the forces. It offers a high level of accuracy, efficiency and easiness of implementation.

In order to use a Kalman filter, it is also required to know the statistical properties of the system (or plant) and sensors, usually represented by the noise covariance matrices. The later is easy to obtain through sensor characterization or through the manufacturer's information. However, the statistical properties of the plant are not straightforward to obtain. Usually, these properties are determined based on a trial-and-error procedure. Nevertheless, there is no guarantee that the values achieved are the ones that lead to the most accurate estimations and, in addition, these values can be also maneuver dependent. Hence, in a different set-up, the statistical properties have to be adjusted again, incrementing the development time and limiting the versatility of the approach.

As a solution, in [2], an adaptive Kalman filter (AKF) was developed for multibody models. It combined the maximum likelihood method [3] for estimating the plant covariance matrix with the errorEKF\_FE, resulting in the AerrorEKF\_FE. Using the innovation sequence of the Kalman filter (the difference between the virtual measurements predictions and the actual measurements), AerrorEKF\_FE is able to estimate the most probable values for the statistical properties of the system. Results showed that the AerrorEKF\_FE converge to the same level of error, despite the initial guessing about the noise covariances. In [2], other methods are suggested, such as the Sage-Husa [4].

The main drawback of the AerrorEKF\_FE is that it assumes that the system is corrupted by white noise. This is something that cannot be guaranteed for all the systems. Therefore, the performance of the AerrorEKF\_FE could be affected by this problem, leading to inaccurate estimations. This problematic is addressed in [5], where the errorEKF\_FE is combined with a shaping filter. This approach includes the low frequency component of the noise system noise as variable to estimate. For that purpose, the noise is modeled following a Markov model fed with white noise. The resulting system (plant increased with the low frequency component of its noise) is therefore corrupted with white noise. However, in this filter is required to manually adjust the statistical properties of the new system.

This work starts first by evaluating an alternative adaptive method for the AerrorEKF\_FE: the Sage-Husa algorithm. It is of interest to analyze its performance and evaluate if it can improve the performance of the maximum-likelihood method. Later, the AerrorEKF\_FE is combined with a shaping filter (AerrorEKF\_FE\_Sh), so that the adaptive filter can be applied to any system, regardless if it is corrupted by white noise or not. The AerrorEKF\_FE will hence estimate the covariance matrix of the system, which in this version is composed by the multibody system and its noise [5]. Following this approach, the issue of manually adjusting the covariance matrix of the system can be overcome, reducing the development time of a Kalman filter for a particular system, and increasing its versatility.

### 2 Methodology

Following the work of [1], each approach is tested in a four-bar linkage modeled in natural coordinates and using the augmented Lagrangian of index-3 (ALI3P) formulation [6]. All the test are executed in a simulation environment. The *three-simulation method* is followed: three multibody models are developed, so that the first one is considered as the *real mechanism*; the second one acts as a *model* of the *real mechanism*; and the third model is the *model* combined with the proposed filter, which would correct the errors based on the information provided by the measurements taken from the *real mechanism*. In order to replicate a realistic scenario, a modeling error is introduced in the *model*: the gravity force is of  $8.81 \text{ m/s}^2$ , while in the *real mechanism* is of

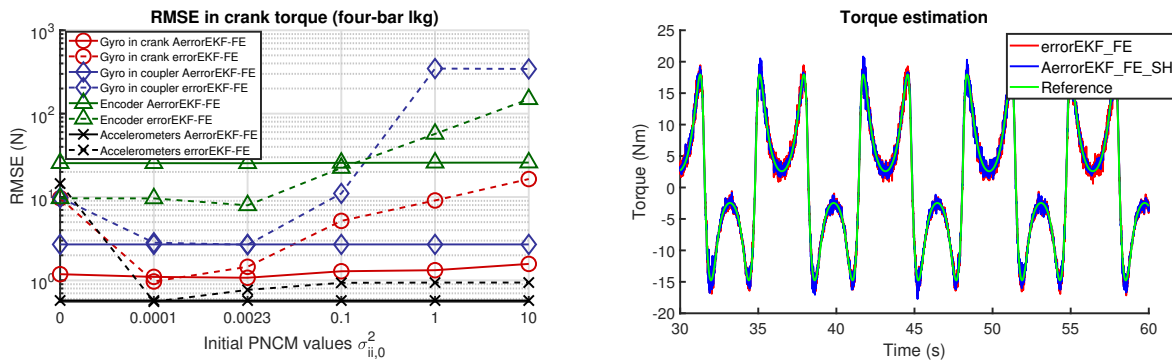
$9.91 \text{ m/s}^2$ . In addition, different sensor configurations for both mechanisms are also tested: an encoder on the crank, a gyroscope on the crank, a gyroscope in the coupler of the mechanism and, finally, a pair of accelerometers at the end of the crank.

In order to test the performance of the AerrorEKF\_FE using the Sage-Husa algorithm for the adaptive part, several simulations are executed. Each simulation is launched under different initial noise covariance matrices. It is expected that the filter converges to a similar accuracy level independently from the initial noise covariance matrices, due to the adaptive equations.

The accuracy of the AerrorEKF\_FE\_Sh is evaluated comparing the accuracy of its estimations the best estimations of the errorEKF\_FE. It is worth remarking that the AerrorEKF\_FE was not able to improve the best estimations of the errorEKF\_FE, as can be seen in [5]. Although it achieved autonomously high accuracy in spite of the initial estimations of the plant noise covariances, it was not able to improve the best result of the errorEKF\_FE, obtained only with one particular value of the plant noise covariance and by trial and error.

### 3 Results

The results are shown in Figure 1a and Figure 1b. Regarding the performance of the AerrorEKF\_FE with the Sage-Husa algorithm, it can be seen how the AerrorEKF\_FE converges to the same level of accuracy despite the initial value assumed for the covariance matrix. Figure 1b shows the torque estimated in the four-bar linkage when the mechanism is instrumented with a pair of accelerometers at the end of the crank. As can be seen, the proposed filter, AerrorEKF\_FE\_Sh, increases the accuracy of the best estimations of the errorEKF\_FE, improving therefore the results of the AerrorEKF\_FE.



(a) Torque estimation error provided by the errorEKF\_FE and the AerrorEKF\_FE combined with the Sage-Husa algorithm.

(b) Torque estimated by the errorEKF\_FE and AerrorEKF\_FE\_Sh compared with the real value (Reference).

It can be concluded that the addition of the shaping filter can help to increase the accuracy of the filter. This can be explained by the fact that the system was not corrupted by white Gaussian noise, fulfilling the initial assumption of the AerrorEKF\_FE.

Using the AerrorEKF\_FE\_Sh, the tedious process of tuning the covariance noise of the system is simplified, leading to a more versatile solution with a reduced development time. The filter can be automatically configured, without requiring a high knowledge of the system. This opens, for example, a possibility for using virtual sensors in applications with high variability, since the Kalman filter will adapt itself to the changes suffered by the system, removing the necessity of configuring manually the filter.

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