Sensitivity analysis of a natural coordinates FFR formulation

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EXTENDED ABSTRACT

1 Introduction

The simulation, analysis and optimization of multibody systems is nowadays facing new and highly computationally demanding problems. The presence of light-weight and fast-moving components in machines and mechanisms has draw the attention of many researches to a proper incorporation of flexibility effects in multibody dynamics (MBD).

Basically, elastic deformation can be included in a dynamic simulation by means of two methods: moving or Floating Frame of Reference methods (FFR) and Inertial Coordinate methods [1]. The first group is characterized by the decomposition of the general motion of a body into a rigid body motion (related to the moving frame) and a flexible deformation with respect to a local undeformed configuration. FFR methods are widely used in the multibody community for mechanisms involving large displacements and small deformations [2]. In Inertial Coordinate methods, on the contrary, rigid body motion and elastic deformation are considered jointly under global displacements. For large elastic deformations, the latest methods are more accurate than FFR.

The Floating Frame of Reference formulation is frequently combined with modal reduction methods, founded on the substitution of flexible local deformations by a collection of mode shapes of each flexible body.

The generation of the Equations of Motion (EoM) of a flexible multibody system in the framework of the FFR method can be addressed with different multibody formulations, regarding which system of rigid body coordinates is considered and how kinematic constraints are enforced. In [3], a natural coordinates Augmented Lagrangian Index-3 formulation with velocity and acceleration projections (ALI3-P) for flexible bodies in the framework of the FFR method was presented, delivering excellent results in terms of accuracy and efficiency. This formulation is revisited in this work from a more comprehensive perspective in order to be implemented as a general formulation in the general purpose multibody library MBSLIM [4].

The optimization of the dynamics of flexible multibody systems is in these days an open subject of continuous study. For gradient based optimization methods, a sensitivity analysis of the dynamic response of the model is mandatory. The present work is devoted as well to the development, implementation and testing of the sensitivity analysis of the revisited multibody formulation for flexible bodies by means of the direct differentiation method.

2 Flexible dynamic formulation

Let us consider the global position of a point P belonging to an elastic body i as:

$$\mathbf{r}_P^i = \mathbf{r}_0^i + \mathbf{A}^i\left(\bar{\mathbf{r}}_{P(r)}^i + \bar{\mathbf{r}}_{P(f)}^i\right) = \mathbf{r}_0^i + \mathbf{A}^i\left(\bar{\mathbf{r}}_{P(r)}^i + \bar{\mathbf{\Psi}}^i\mathbf{q}_f\right) \tag{1}$$

wherein $\mathbf{r}_P^i$ represents the position of P in the global reference frame, $\mathbf{r}_0^i$ denotes the global position of the reference point of body i, $\mathbf{A}^i$ is the rotation matrix of body i with respect to the global reference frame, $\bar{\mathbf{r}}_{P(r)}^i$ is the local rigid body coordinates of point P in body i and $\bar{\mathbf{r}}_{P(f)}^i$ is the deformed part of point P in the local reference frame of body i, which can be evaluated by means of modal reduction as the product of a set of dynamic or static modes ($\bar{\mathbf{\Psi}}^i$) by their amplitudes ($\mathbf{q}_f$).

Taking (1) into account, the dynamics of a multibody system involving flexible bodies can be obtained with an ALI3-P scheme:

$$\left[\mathbf{M}\ddot{\mathbf{q}}^* + \mathbf{K}\mathbf{q} + \mathbf{\Phi}_\mathbf{q}^\mathrm{T}\left(\boldsymbol{\lambda}^* + \boldsymbol{\alpha}\mathbf{\Phi}\right)\right]^{\{i\}} = \left[\mathbf{Q}_e - \dot{\mathbf{M}}\dot{\mathbf{q}} + \frac{1}{2}\left(\dot{\mathbf{q}}^\mathrm{T}\mathbf{M}_\mathbf{q}\dot{\mathbf{q}}\right)^\mathrm{T}\right]^{\{i\}} \tag{2a}$$

$$\boldsymbol{\lambda}^{*\{i+1\}} = \boldsymbol{\lambda}^{*\{i\}} + \boldsymbol{\alpha}\mathbf{\Phi}^{\{i\}};\quad i = 0,1,2,... \tag{2b}$$

being $\mathbf{q}$ the set of rigid and flexible coordinates, $\mathbf{\Phi}$ the vector of kinematic constraints, $\mathbf{M}$ the mass matrix, $\mathbf{Q}_e$ the vector of external forces, $\mathbf{K}$ the stiffness matrix accounting for flexibility, $\ddot{\mathbf{q}}^*$ the unprojected accelerations, $\boldsymbol{\lambda}$ the approximate Lagrange multipliers, $\boldsymbol{\alpha}$ a diagonal penalty matrix and the superscript i indicates the iteration index. Moreover, $\dot{(\cdot)}$ represents a first derivative with respect to time, $\ddot{(\cdot)}$ a second temporal derivative and subscripts denote partial derivatives.

Equations (2) can be stabilized through projections onto the velocity and acceleration constraint manifolds with:

$$\left(\bar{\mathbf{P}} + \varsigma\mathbf{\Phi}_\mathbf{q}^\mathrm{T}\boldsymbol{\alpha}\mathbf{\Phi}_\mathbf{q}\right)\dot{\mathbf{q}} = \bar{\mathbf{P}}\dot{\mathbf{q}}^* - \mathbf{\Phi}_\mathbf{q}^\mathrm{T}\varsigma\boldsymbol{\alpha}\mathbf{\Phi}_t \tag{3}$$

$$\left(\bar{\mathbf{P}}+\varsigma\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\alpha}\mathbf{\Phi}_{\mathbf{q}}\right)\ddot{\mathbf{q}}=\bar{\mathbf{P}}\ddot{\mathbf{q}}^{*}-\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\varsigma\boldsymbol{\alpha}\left(\dot{\mathbf{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}}+\dot{\mathbf{\Phi}}_{t}\right) \tag{4}$$

where $\bar{\mathbf{P}}$ is a symmetric projection matrix and ς is a penalty factor.

3 Sensitivity analysis

Taking derivatives on equations (2), (3) and (4), a forward sensitivity formulation can achieved in accordance to the process and structure presented in [5]. The complexity at this point emerges from the evaluation of the derivatives of each flexible dynamic term present in the dynamic equations.

Both dynamic and sensitivity equations have been implemented in the multibody library MBSLIM as general dynamic and sensitivity formulations. For the preliminary modal analysis, the PDE tool of Matlab have been considered.

4 Results

The general implementation allows the simulation and optimization of a wide variety of mechanisms, among which we have selected the slider crank mechanism with a flexible rod described in [3] as testing example for the dynamics and sensitivities.

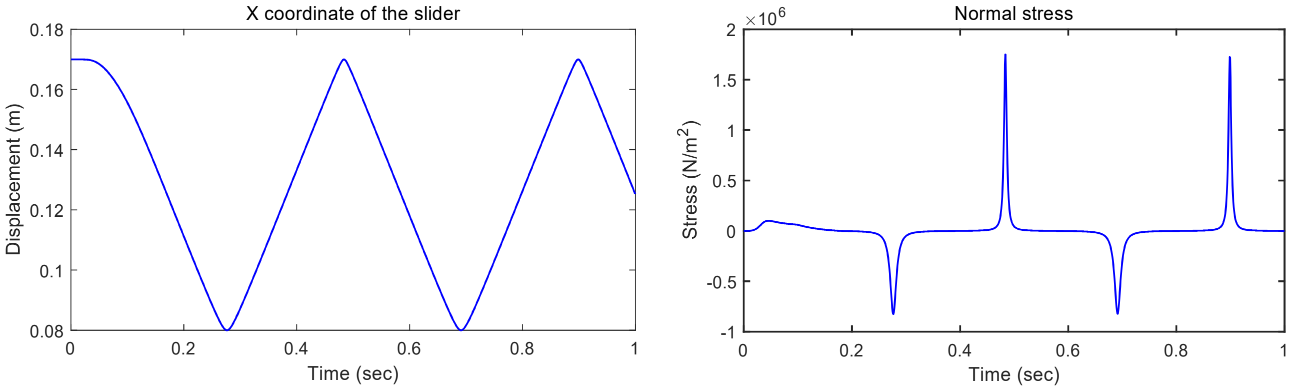

Figure 1: Displacement of the X coordinate of a point of the slider (on the left) and normal stress at the center of the middle section of the rod (on the right).

In figure 1, the results of the evaluation of the dynamics with the general code implemented in MBSLIM are showed, and they display a great level of convergence with the results included in [3] for the same mechanism. For the sensitivity analysis validation, different objective functions and parameters have been considered.

Acknowledgments

The support of the Spanish Ministry of Science and Innovation (MICINN) under project PID2020-120270GB-C21 is greatly acknowledged.

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