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Coupled design and control of a tilting tricycle.

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Abstract

The kinematic and dynamic optimization of complex multibody systems opens the possibility of enhancing the design of novel and existing vehicles. In this work, the sensitivity analysis and optimization of general multibody systems is described under kinematics and dynamics conditions and applied to the optimization of a tilting three wheeler. The approach proposed is general and valid for any multibody system, because the general kinematics and dynamics sensitivity equations are the starting point. The implementation of the equations and the numerical experiments have been built in the MBSLIM multibody library coded mostly in Fortran 2008.

1 Introduction

The three-wheeled tilting vehicle shown in Figure 1 is an alternative to common bicycles in which the front wheel and fork are replaced by two front wheels mounted in knuckles, driven by a 1 degree of freedom (DOF) tilting mechanism and a 1 DOF steering system, similar to those employed in four-wheeled vehicles. With these modifications the same driving principle of a classic bicycle keeps for the tricycle, because the front wheels are able to rotate around their axles and the frame is able to roll freely. Nevertheless the design of the tilting and steering mechanisms of the tricycle is not straightforward: first, the steering system must approximately satisfy Ackerman's steering condition for any combination of inclination of the frame and rotation of the handlebar; second, additional kinematic or dynamic conditions can be of interest, for example, achieving a desired relation between the handlebar rotation and the heading of the wheels, or minimizing the coupling between the brakes forces and the steering.

Four optimization problems have been solved: three optimal design problems solved sequentially, one kinematic and two dynamic, to compare the solutions and one optimal control problem, required for the dynamics of the vehicle and therefore solved together with the dynamic design optimization problems.

All the optimizations performed are gradient-based and rely on the multibody general sensitivity equations implemented in MBSLIM which can handle design and control parameters at the same time, allowing the co-design by combining optimal design and optimal control together.

2 Optimization problem statement

Let us consider a multibody system modeled with $\mathbf{q} \in \mathbb{R}^n$ coordinates. The configuration of the system as a function of time, $\mathbf{q}(t, \boldsymbol{\rho})$, is given by the solution of the kinematics or dynamics equations and it is considered dependent on a set of parameters $\boldsymbol{\rho} \in \mathbb{R}^p$, which can be geometric parameters in the case of kinematic optimization but also parameters affecting forces or masses in the case of dynamic optimization. Consider an objective function $\psi \in \mathbb{R}$, expressed as an integral over time:

$$\psi = \int_{t_0}^{t_F} g(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\rho}) \, \mathrm{dt}. \tag{1}$$

The optimization problem consist in calculating the set of parameters which minimize the objective function subject to the optimization constraints $\boldsymbol{\rho}^* = \operatorname{argmin}(\psi)$.

The sensitivity analysis of the objective function and constraints with respect to the set of parameters can be computed by means of the direct sensitivity or adjoint sensitivity methods and using the kinematic or the dynamic equations presented before [1, 2].

As numerical optimizers, an in-house augmented Lagrangian optimizer available in MBSLIM has been tested against two third-party tools, a Fortran implementation of the LBFGS algorithm and the *MATLAB fmincon* algorithms.

3 Numerical experiments

The case study for optimal design and optimal control is the tilting three wheeled vehicle shown in Figure 1. The optimal design can be accomplished by means of a kinematic analysis or by means of a dynamic analysis in order to better optimize for the service conditions of the vehicle.

Three following optimization problems have been solved for the design of the steering:

1. Kinematic design optimization with an objective function imposing the satisfaction of the Ackerman's condition and a kinematic maneuver, covering a range of tilting and steering angles. For the kinematic simulation, the degrees of freedom of the vehicle are predetermined and the optimization is carried out over this prescribed motion, but for the dynamic simulation, an optimal control function will be added to force the vehicle to fit the desired trajectory and speed, controlling the handlebar and pedals.

2. Dynamic design optimization with the same objective function and a double lane change maneuver.

3. Dynamic design optimization with the same maneuver, but an objective function minimizing the energy that the rider has to introduce in the system through the pedals.

4. Since the two design optimizations proposed in 2. and 3. are accomplished under dynamic conditions, a double lane change maneuver is imposed using optimal control for the handlebar and pedals of the vehicle in order to accomplish the maneuver while keeping the stability of the tricycle.
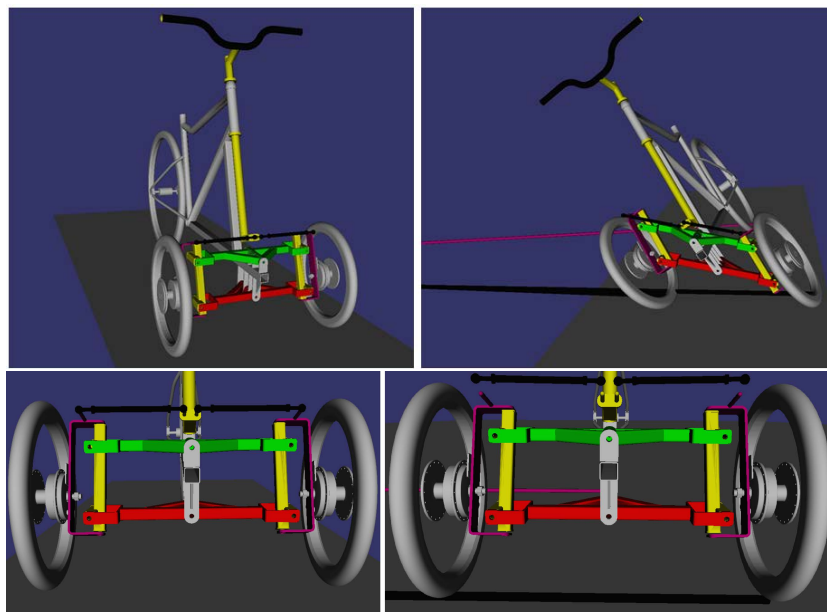

Figure 1: Three wheeled tilting vehicle (up); unoptimized vs. optimized design (bottom)

4 Results and conclusions

The present work proposes an approach for enhancing the design of mechanical systems using kinematics or dynamics optimizations. The approach is applied to the optimal design and control of a three-wheeled tilting vehicle resulting in an improvement of the design while keeping the control of the system.

References

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