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Effectiveness, Robustness, and Applicability of Formulations for Multibody Dynamics: A Comparative Study

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Abstract

Several different methods and formulations have been proposed for the simulation of multibody systems. In this work, the focus is placed on examining the effectiveness of representative formulations by employing well-established benchmark models and evaluation methods. Specifically, four (4) different formulations have been used to establish the equations of motion (EOMs) and the kinematic constraints of the mechanical systems, namely a recently proposed Ordinary Differential Equations (ODE) formulation, the penalty formulation, an augmented Lagrangian algorithm, and a stabilized Lagrangian approach. Concerning the benchmark models, a nonlinear model of a single planar pendulum is initially examined, followed by a slider-crank mechanism. The dynamics equations have been integrated by employing two (2) fixed-step formulas, namely the forward-Euler (FE) method and the trapezoidal rule (TR). Subsequently, a full set of numerical results have been extracted in order to assess the effectiveness, robustness, and applicability of these formulations. Lastly, the influence of the formulation parameters has also been examined and discussed.

Introduction

In general, the field of multibody dynamics deals with systems composed of several rigid or flexible bodies, which are constrained through kinematic joints. The EOMs and the constraint equations are, usually, expressed as a set of Differential-Algebraic Equations (DAEs). During the last decades, numerous methods and formulations have been presented for solving such systems in DAEs form. However, different approaches have also been proposed which express the EOMs and the constraint equations of multibody systems as a set of pure Ordinary Differential Equations. Within this work, the focus is placed on evaluating the effectiveness of various multibody formulations by simulating well-established benchmark models.

Multibody formulations and benchmark models

Herein, four (4) different formulations have been employed for performing the forward-dynamics simulation of the examined models. First, a recently proposed formulation [1] is examined, which leads to a set of second order ordinary differential equations for the equations of motion and the constraint equations. This natural ODE formulation stems from a physically consistent description of the dynamics of constrained mechanical systems. A prominent feature of the resulting set of equations lies in the presence of inertia, damping, and stiffness terms in both the EOMs and the constraint equations. These terms enable the natural scaling and stabilization of these equations.

The remaining three (3) formulations have already been extensively used during the last decades. In particular, the second method is the penalty formulation presented in [2], which replaces the kinematic constraints $\underline{\phi} = \underline{0}$ with penalty mass-spring-damper systems. Moreover, the augmented Lagrangian formulation proposed in [2] as solution for the limitations of the above-mentioned penalty approach is also employed here. The last method is a stabilized Lagrangian approach, based on the classical Baumgarte stabilization method [3]. The numerical integration for all formulations is performed by utilizing two (2) well-established fixed-step methods, namely the forward-Euler method and the trapezoidal rule. It should also be mentioned that the code implementation was performed in MATLAB for all examined cases.

Two (2) example models have been used for assessing the effectiveness of the above-described formulations, namely a planar pendulum and a slider-crank mechanism. The first example is a classical, nonlinear model which moves under the effect of gravity. Since this system is conservative, the mechanical energy error can be examined in order to assess the validity of the simulation results, besides comparing the time history of the position variables to a reference solution. The second model is a variation of the well-known slider-crank mechanism, presented in [4]. For this example, two different cases, with and without the application of an external force, were examined. Again, a proper reference solution is used to evaluate the validity of the numerical results obtained by the different formulations. It is worth noting that the reference solution for both benchmark models was derived through the convergence of different simulation methods. The accuracy of each solution method is evaluated through the comparison of aggregates of the time-history of the difference between the reference solution and simulation results for selected system variables, as reported in [4]. Different methods are then compared in terms of their computational efficiency for equivalent accuracy levels.

Preliminary results

Initial results with the selected formulations and test problems confirmed that it is possible to establish a comparison framework using reference solutions obtained at convergence.

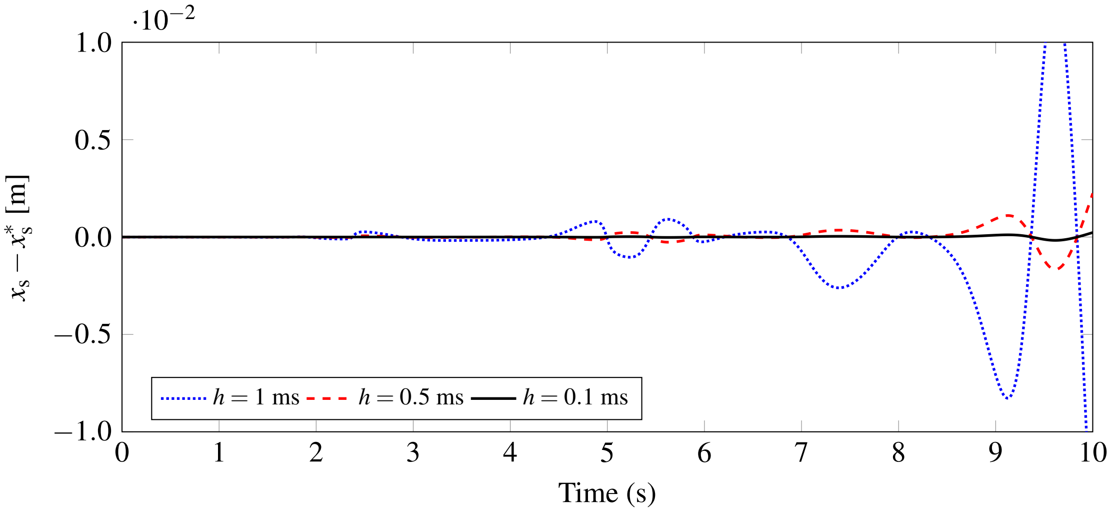

Figure 1: Convergence of the solution delivered by the formulation in [1] towards the reference solution defined in [4] for different values of the integration step-size.

Figure 1 displays the difference between the x-coordinate of the slider-crank example delivered by the numerical simulation, x_s, and the reference solution reported in [4], x_s^*. Results converge towards the reference solution, enabling the comparison of the times elapsed in computations with different methods for equivalent levels of accuracy. The admissible values of the formulation parameters, e.g., penalty and stabilization factors, can also be determined by varying their values within test ranges and verifying if the results achieve the specified tolerance values, and how the changes affect the computational performance.

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