30th International Congress on Sound and Vibration

ICSV30 AMSTERDAM 8-11 JULY 2024

USING EIGENSTRUCTURE ASSIGNMENT FOR SPILLOVER COMPENSATION IN EXPLICIT CO-SIMULATION OF LINEAR VIBRATING SYSTEMS

Dario Richiedei, Iacopo Tamellin*
University of Padova, Vicenza, Italy
email: iacopo.tamellin@unipd.it

Francisco González
University of A Coruña, Ferrol, Spain

Eigenstructure assignment is a widely used technique to impose the desired natural frequencies and mode shapes of vibrating systems through structural modification or active control (such as state feedback). This approach has been effectively used in time delayed systems where delays perturb the system poles deteriorating the overall system stability and performances. The undesired shift of the system poles is referred to as spillover. In this work, eigenstructure assignment is used for the correction of the co-simulation which is an emerging technique to determine the dynamic behaviour of complex engineering applications. In this simulation paradigm, the overall system dynamics is split into several subsystems that evolve in time separately. This enables to use ad-hoc solution methods which are tailored for the specific nature of the subsystem to be simulated. The overall system dynamics is obtained through the exchange of information between subsystems to provide the inputs for the simulation of the others. This process is handled through a discrete-time interface that manages the co-simulation performing the numerical extrapolation of the data. In this light, the limited exchange of data can cause accuracy and stability issues in the simulation leading to inaccurate results. This work provides an analysis of this critical problem, revealing that coupling subsystems through a co-simulation interface introduces a time-delay in the exchange of data. This makes the poles of the co-simulated system to spillover altering their locations with respect to those of the original system to be simulated. Then, a correction algorithm based on the paradigm of eigenstructure assignment is proposed to compensate the problem. The effectiveness of the proposed method is shown through its application to the explicit co-simulation of a meaningful linear vibrating system. The results show that the proposed method can effectively correct the co-simulation.

Keywords: eigenstructure assignment, co-simulation, explicit coupling schemes, vibrating systems.

1. Introduction

Co-simulation is a growing simulation paradigm adopted in the last two decades to predict the dynamic behaviour of dynamical systems by dividing the system into more subsystems. This approach yield the main advantage of tailoring each simulator to the specific subsystem which is going to be simulated [1]. Then, each subsystem exchange information at discrete instants by means of coupling variables. The handling of these coupling variables performed by a co-simulation manager which provides the input signals to the subsystems.

Co-simulation setups can be implicit (iterative) or explicit (noniterative) [2]. The first are more stable and accurate. While the second suffer from instability and inaccuracy. However, explicit schemes are more prone for real-time applications. In this light, explicit co-simulation schemes are widely adopted.

Several methods have been proposed in the literature to mitigate the above mentioned problems of explicit co-simulation setups such as: input extrapolation [3], step-size modification [4] or energy monitoring and compensation [5]. Recently, compensation schemes have been introduced to modify the coupling variables to remove the perturbation introduced by the co-simulation interface (see e.g. [6]).

This paper proposed a model of explicit Jacobi co-simulated systems, evidencing the eigenstructure spillover due to the co-simulation interface. Then, an eigenstructure assignment method is developed to compensate for the perturbation due to the co-simulation manager. Eigenstructure assignment is a control technique widely adopted in the field of vibration control (see e.g. [7,8]), it consists of tuning the controller ensuring the assignment of the desired eigenvalues and eigenvectors.

The effectiveness of the proposed method is assessed through co-simulations performed on a motor-gearbox-load with viscoelastic coupling model.

2. On the co-simulation of mechanical systems

2.1 Model of the monolithic system

The system studied in this paper consists of a motor, coupled to a constant inertial load through a flexible gearbox, as sketched in Figure 1. Two off-the-shelf components are assumed: a Rockwell MPL-B230P servomotor and a Wittenstein SP075 planetary gearbox. The system features a spring at the motor side whose stiffness is $k_T = 4.24$ Nm/rad and $c_T = 1\text{e-}5$ Nms/rad is the motor viscous friction. Accordingly with the manufacturer specification, a linear viscoelastic model is adopted for the gearbox, by means of a load-side spring, whose stiffness is $k = 3.5\text{e}4$ Nm/rad, and a damper whose damping ratio is $c = 1\text{e-}5$ Nms/rad. The motor rotor moment of inertia is $J_M = 6.3\text{e-}5$ kgm^2, the moment of inertia of the gearbox with respect to the motor shaft is $J_G = 1\text{e-}5$ kgm^2 . The load constant moment of inertia is $J_L = 1.025$ kgm^2. The gear ratio is $\tau = 1/32$; however, due to flexibility, it just represents the low-frequency transmissibility from motor speed $\dot{\theta}_M$ to load speed $\dot{\theta}_L$. Hence, a two degrees of freedom model should be adopted:

$$\begin{cases} \left(J_M + J_G\right)\ddot{\theta}_M\left(t\right) = T_M\left(t\right) - \tau T_C\left(t\right) \\ J_L\ddot{\theta}_L\left(t\right) = T_C\left(t\right) \end{cases} \tag{1}$$

where T_M is the motor torque and T_C is the coupling torque which can be computed, due to the viscoelastic coupling, as:

$$T_C\left(t\right) = -k\left(\theta_L\left(t\right) - \tau\theta_M\left(t\right)\right) - c\left(\dot{\theta}_L\left(t\right) - \tau\dot{\theta}_M\left(t\right)\right) \tag{2}$$

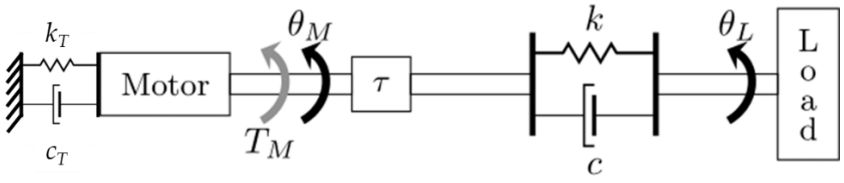

Figure 1: The motor-gearbox-load system with viscoelastic coupling.

The presence of a torque T_M does not affect eigenstructure and hence can be discarded for the purposes of the paper.

Accordingly to these definitions, the monolithic system model in the Laplace domain $s = j\omega$, is:

$$\begin{cases} s^2\left(J_M+J_G\right)\theta_M(s) = -\left(sc_T+k_T\right)\theta_M(s) + \tau\left(sc+k\right)\left(\theta_L(s)-\tau\theta_M(s)\right) \\ s^2 J_L\theta_L(s) = -\left(sc+k\right)\left(\theta_L(s)-\tau\theta_M(s)\right) \end{cases} \tag{3}$$

and it can be recast in the compact matrix form into:

$$\mathbf{G}_{\mathrm{t}}(s)\boldsymbol{\theta}(s) = \boldsymbol{0} \tag{4}$$

with:

$$\mathbf{G}_{\mathrm{t}}(s) = s^2\begin{bmatrix} J_M+J_G & 0 \\ 0 & J_L \end{bmatrix} + s\begin{bmatrix} \tau^2 c + c_T & -\tau c \\ -\tau c & c \end{bmatrix} + \begin{bmatrix} \tau^2 k + k_T & -\tau k \\ -\tau k & k \end{bmatrix},\ \boldsymbol{\theta}(s) = \begin{Bmatrix} \theta_M(s) \\ \theta_L(s) \end{Bmatrix} \tag{5}$$

2.2 Model of the co-simulated system

In co-simulation setups the coupling variables are exchanged through a co-simulation manager at each communication point [1]. Then, the co-simulation manager adjusts the inputs exchanged the subsystems by extrapolating them from the values provided at previous communication points. The time span between two consecutive communication points, in the case of single rate co-simulation, is T_s. In this paper T_s is assumed to be equal to 0.1 ms.

In the case of force-displacement coupling, as sketched in Figure 2, a representation in the s-domain can be adopted to model the extrapolation operation of the co-simulation manager as follows:

$$T_C^*(s) = D(s)T_C(s), \quad \theta_L^*(s) = D(s)\theta_L(s) \tag{6}$$

In this work the extrapolation algorithm used by the co-simulation manager is the zero-order hold (ZOH), whose transfer function is [10]:

$$D(s) = \frac{1-e^{-sT_s}}{sT_s} \tag{7}$$

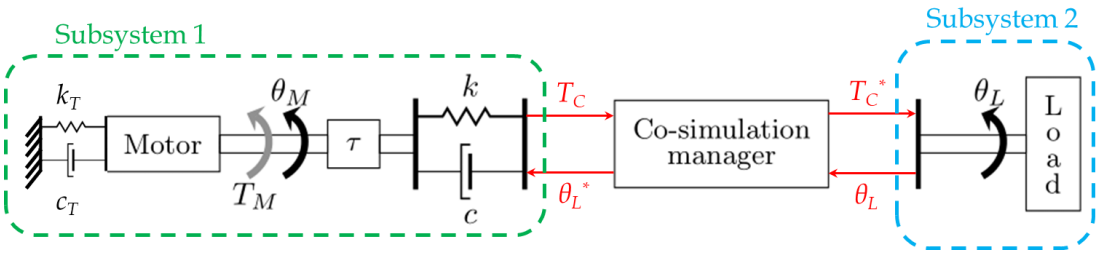

Figure 2: Sketch of the co-simulated system.

In the co-simulation environment Eq.(3) can be adapted accordingly to Eq.(6), as follows:

$$\begin{cases} s^2\left(J_M+J_G\right)\theta_M(s) = -\left(sc_T+k_T\right)\theta_M(s) + \tau\left(sc+k\right)\left(D(s)\theta_L(s)-\tau\theta_M(s)\right) \\ s^2 J_L\theta_L(s) = -D(s)\left(\left(sc+k\right)\left(D(s)\theta_L(s)-\tau\theta_M(s)\right)\right) \end{cases} \tag{8}$$

Eq.(8) can be manipulated to provide a convenient formulation of the co-simulated system model in the Laplace domain as follows:

$$\left(\mathbf{G}_{\mathrm{t}}(s) + \begin{bmatrix} 0 & \tau\left(sc+k\right)\left(1-D(s)\right) \\ \tau\left(sc+k\right)\left(1-D(s)\right) & \left(sc+k\right)\left(D^2(s)-1\right) \end{bmatrix}\right)\boldsymbol{\theta}(s) = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{9}$$

The comparison of the monolithic model in Eq.(4) with the co-simulated model in Eq.(9) shows that the co-simulation interface introduces a perturbation whose form is:

$$\mathbf{\Delta G}_c(s)=\begin{bmatrix} 0 & \tau(sc+k)(1-D(s)) \\ \tau(sc+k)(1-D(s)) & (sc+k)(D^2(s)-1) \end{bmatrix} \tag{10}$$

3. Eigenstructure spillover and the compensation algorithm

3.1 Eigenstructure spillover

The eigenproblem related to Eq.(4) is defined by the quadratic matrix pencil:

$$\mathbf{G_t}(\lambda_i)\boldsymbol{w_i}=\boldsymbol{0} \tag{11}$$

Each eigenpair $(\lambda_i, \boldsymbol{w_i})$ is composed by the *i*-th eigenvalue λ_i and the *i*-th eigenvector $\boldsymbol{w_i}$, together the *2N* eigenpairs (where *N* is the number of DOFs of the system) compose the eigenstructure of the system. Eigenvalues and eigenvectors are both fundamental to describe the dynamic behaviour of the dynamic system. In particular, in the case of vibrating systems as the one considered in this work, eigenvalues appears in complex conjugate pairs and define the natural frequencies and the damping, as a consequence also the settling time and the speed of response. Simultaneously, the eigenvectors define the spatial shape of vibration as well as the eigenvalue sensitivity to the model parameters.

In co-simulation the quadratic matrix pencil can be inferred from Eq.(9) and it is the following:

$$\left(\mathbf{G_t}(\tilde{\lambda}_i)+\mathbf{\Delta G}_c(\tilde{\lambda}_i)\right)\tilde{\boldsymbol{w}}_i=\boldsymbol{0} \tag{12}$$

The eigenproblem in Eq.(12) is transcendental [11] due to the presence of the exponential terms introduced by *D*(*s*). Transcendental eigenproblems feature *2N* primary roots, the "physical ones", and an infinite number of secondary or latent roots. It is fundamental to observe that the theoretical monolithic eigenstructure $(\lambda_i, \boldsymbol{w_i})$ is altered into $(\tilde{\lambda}_i, \tilde{\boldsymbol{w}}_i)$ for the co-simulated system. Obviously, altering the eigenstructure of the system results in altering the system evolution, and hence the results of the simulation. The perturbation of the 2*N* primary roots, is known as the so called ''pole spillover'' [12]. Spillover is a threat for two reasons: first, it might shift the primary roots to the right half complex plane, thus destabilizing the co-simulation. Second, even if the shift does not destabilize the system it alters the system dynamics affecting the accuracy of the results obtained through the simulation.

3.2 Co-simulation compensation algorithm

In this light, this work proposes a co-simulation compensation algorithm which aims at countering the perturbation introduced by the co-simulation interface. The algorithm is inspired by the one proposed by the Authors in [13]. A position-velocity-acceleration (PVA) feedback controller is designed as follows:

$$\begin{cases} u_1(s)=-\left(g_1+sf_1+s^2d_1\right)\theta_M(s)-\left(g_2+sf_2+s^2d_2\right)\theta_L^*(s) \\ u_2(s)=-\left(g_3+sf_3+s^2d_3\right)\theta_M^*(s)-\left(g_4+sf_4+s^2d_4\right)\theta_L(s) \end{cases} \tag{13}$$

The compensated co-simulated system dynamic model is obtained by introducing the control forces in Eq.(13) on the right-hand side of Eq.(9) and it becomes:

$$\left(\mathbf{G_t}(s)+\mathbf{\Delta G}_c(s)+\begin{bmatrix} g_1+sf_1+s^2d_1 & \left(g_2+sf_2+s^2d_2\right)D(s) \\ \left(g_3+sf_3+s^2d_3\right)D(s) & g_4+sf_4+s^2d_4 \end{bmatrix}\right)\boldsymbol{\theta}(s)=\boldsymbol{0} \tag{14}$$

Compensation is achieved if the quadratic matrix pencil related to Eq.(14) yields the same eigenstructure of the monolithic one in Eq.(11). Since $\mathbf{G}_\mathbf{t}\left(\lambda_i\right)\boldsymbol{w_i}=\boldsymbol{0}$, the following eigenproblem is obtained:

$$\left(\mathbf{\Delta G}_{\boldsymbol{c}}\left(\lambda_i\right)+\begin{bmatrix} g_1+\lambda_i f_1+\lambda_i^2 d_1 & \left(g_2+\lambda_i f_2+\lambda_i^2 d_2\right)D\left(\lambda_i\right) \\ \left(g_3+\lambda_i f_3+\lambda_i^2 d_3\right)D\left(\lambda_i\right) & g_4+\lambda_i f_4+\lambda_i^2 d_4 \end{bmatrix}\right)\boldsymbol{w_i}=\boldsymbol{0} \tag{15}$$

Eq.(15) is recast into the following linear system with respect to the unknown compensation gains, $\boldsymbol{g}=\left[g_1,...,g_4\right]^T$, $\boldsymbol{f}=\left[f_1,...,f_4\right]^T$ and $\boldsymbol{d}=\left[d_1,...,d_4\right]^T$, to be satisfied simultaneously for $i = 1, \ldots, 2N$:

$$\begin{bmatrix}\mathbf{L_a} & \mathbf{L_v} & \mathbf{L_p}\end{bmatrix}\begin{Bmatrix}\boldsymbol{d}\\ \boldsymbol{f}\\ \boldsymbol{g}\end{Bmatrix}=-\mathbf{\Delta G}_{\boldsymbol{c}}\left(\lambda_i\right)\boldsymbol{w_i},\ \text{with}$$

$$\mathbf{L_p}=\begin{bmatrix}\boldsymbol{w}_{i,1} & D\left(\lambda_i\right)\boldsymbol{w}_{i,2} & 0 & 0\\ 0 & 0 & D\left(\lambda_i\right)\boldsymbol{w}_{i,1} & \boldsymbol{w}_{i,2}\end{bmatrix},\quad \mathbf{L_v}=\lambda_i\mathbf{L_p},\quad \mathbf{L_a}=\lambda_i^2\mathbf{L_p} \tag{16}$$

3.3 Application of the algorithm

The eigenstructure of the monolithic system is compared with the one of the co-simulated system without compensation in Table 1. The exponential terms in $D(s)$ are computed through the 6th order Padé approximation [14]. Two unstable pole pairs $\tilde{\lambda}_{1,2}$ and $\tilde{\lambda}_{3,4}$ are detected, i.e., the co-simulation (without compensation) will be unstable. The application of the compensation algorithm proposed yields $\boldsymbol{g}$=[-7.05e-4,-2.2e-5,-0.045,-0.0014]^T, $\boldsymbol{f}$=[-2.9e-7,-0.055,-0.055,3.499]^T and $\boldsymbol{d}$=[-1.2e-8,-6.2e-6,-1.7e-6,-5.5e-4]^T. The eigenvalues and eigenvectors of the compensated co-simulated system match with those of the monolithic system as desired. The eigenvalues of the monolithic and co-simulated systems in the complex plane are shown in Figure 3.

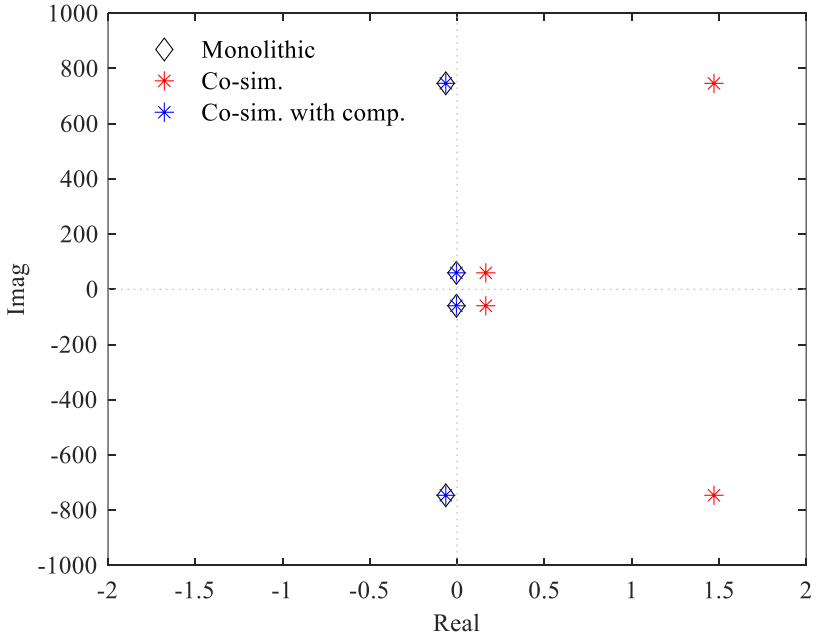

Figure 3: Dominant eigenvalues of the monolithic and co-simulated system without and with compensation.

Table 1: Eigenstructure of the monolithic and co-simulated systems.

$\lambda_{1,2}$	$\boldsymbol{w}_{1,2}$	$\tilde{\lambda}_{1,2}$	$\tilde{\boldsymbol{w}}_{1,2}$
-0.0038 ± 59.678i	5.024e-5 ∓ 0.99939i	0.1644 ± 59.677i	∓0.99939i
	2.268e-6 ∓ 0.034868i		0.00010589 ± 0.034868i
$\lambda_{3,4}$	$\boldsymbol{w}_{3,4}$	$\tilde{\lambda}_{3,4}$	$\tilde{\boldsymbol{w}}_{3,4}$
-0.0648 ± 746.250i	-1	1.4718 ± 746.180i	-1
	0.0020413 ∓ 3.770e-7i		0.0020392 ± 7.8e-5i

4. Co-simulation example

The proposed algorithm is applied to the co-simulation setup previously discussed. The subsystem integration schemes are explicit Jacobi [6], i.e., no rollover is admitted during the integration, this is the typical scenario of real-time applications. A first-order semi-implicit Euler integrator is adopted [6].

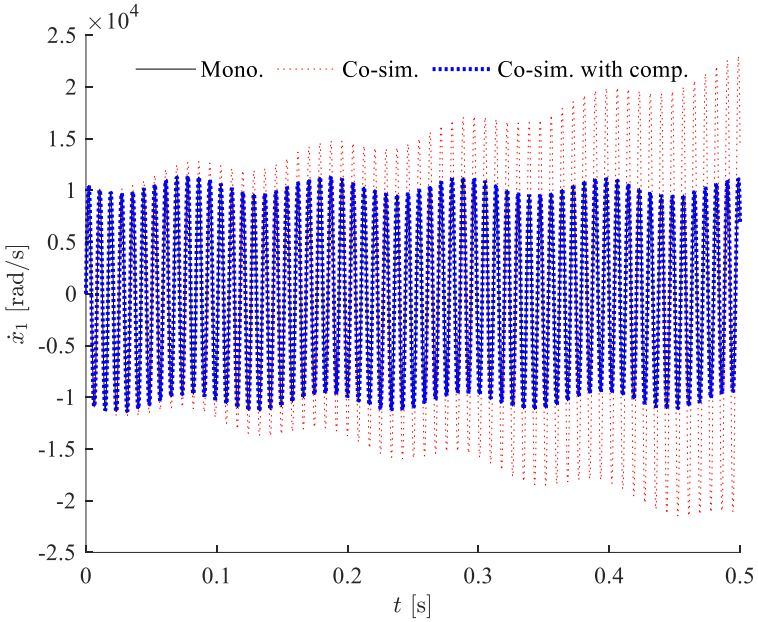

Figure 4: Motor speed: monolithic and co-simulation results.

The monolithic and the co-simulated system without and with the proposed compensation method are simulated in the free-evolution from the sample initial conditions $\boldsymbol{\theta}(0) = \left[0, \frac{\pi}{6}\right]^T$, $\dot{\boldsymbol{\theta}}(0) = [0,0]^T$ and the simulation time is set to 0.5 s. The results of the monolithic system are taken as reference. The speed of the motor and of the load are shown respectively in Figure 4 and Figure 5, while the overall mechanical energy of the system is shown in Figure 6. The results here provided clearly show the effectiveness of the proposed compensation algorithm. Indeed, the co-simulation without compensation is unstable. This result is expected as discussed in Section 3 and is related to the eigenstructure spillover introduced by the co-simulation interface. The developed eigenstructure assignment compensation algorithm enables

to tackle this aspect and finally the co-simulation is accurate, and the results approach the theoretical monolithic ones.

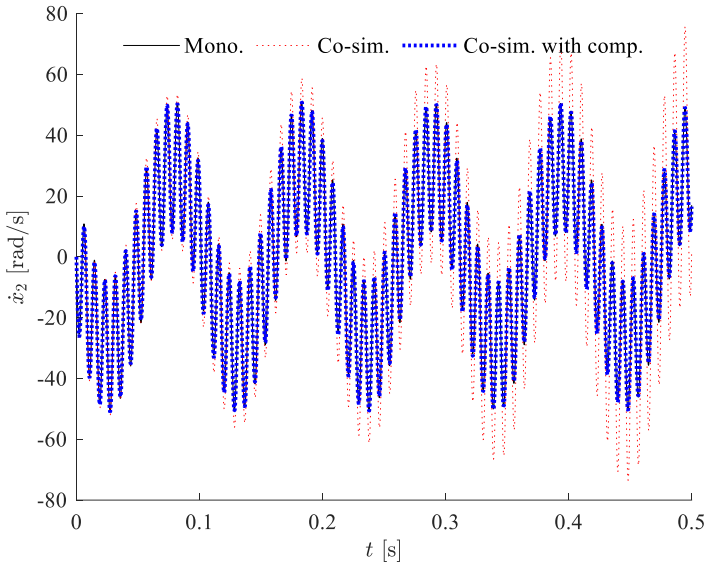

Figure 5: Load speed: monolithic and co-simulation results.

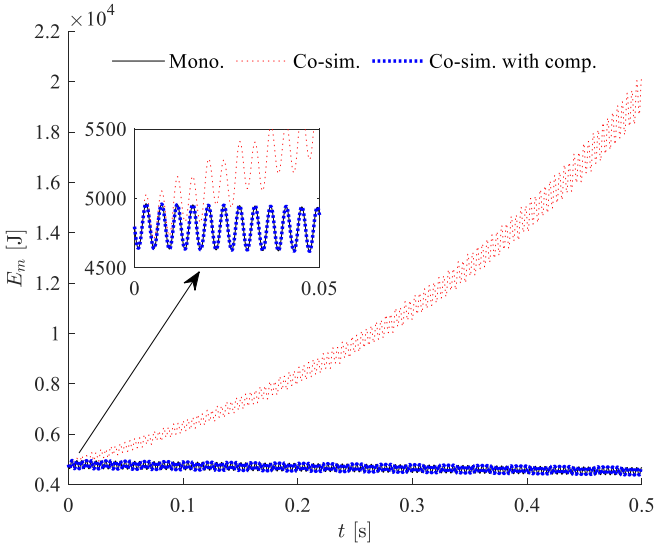

Figure 6: Mechanical energy: monolithic and co-simulation results.

5. Conclusions

This paper proposes a method to evaluate the spillover of the eigenstructure of a co-simulated mechanical system. This phenomenon is critical because it decreases the accuracy of the co-simulation, and can also make the simulation unstable. The perturbation of the eigenstructure is induced by the extrapolation operation performed at the co-simulation interface by the manager to orchestrate the exchange of inputs and outputs between the subsystems composing the co-simulation setup.

In this light, a compensation algorithm is proposed. The algorithm relies on the model of the co-simulation interface and the compensation force is computed by assigning the theoretical monolithic system eigenvalues and eigenvectors through a state-feedback controller.

The theory is discussed for the test-case of a mechatronic system composed by a motor-gearbox-load system characterized by a viscoelastic transmission. The effectiveness of the proposed algorithm is then demonstrated on an explicit co-simulation setup. Indeed, the eigenstructure spillover is correctly detected and then the co-simulation is compensated by adopting the proposed state-feedback compensation method.

REFERENCES

1 Gomes, C., Thule, C., Broman, D., Larsen, P.G and Vangheluwe, H. Co-Simulation: A Survey, *ACM Computing Surveys*, **51** (3), 1-33, (2018).

2 Schweizer, B., Li, P. and Lu, D. Explicit and Implicit Cosimulation Methods: Stability and Convergence Analysis for Different Solver Coupling Approaches, *Journal of Computational and Nonlinear Dynamics*, **10** (5), 051007, (2015).

3 Busch, M. Continuous approximation techniques for co-simulation methods: Analysis of numerical stability and local error, *ZAMM Journal of Applied Mathematics and Mechanics*, **96** (9), 1061-1081, (2016).

4 Benedikt, M., Watzenig, D., Zehetner, J. and Hofer, A. Macro-step-size selection and monitoring of the coupling error for weak coupled subsystems in the frequency-domain*, 5th International Conference on Computational Methods for Coupled Problems in Science and Engineering, COUPLED PROBLEMS 2013,* Ibiza, Spain, (2013).

5 Sadjina, S., Kyllingstad, S., Skjong, S. and Pedersen, E. Energy conservation and power bonds in co-simulations: non-iterative adaptive step size control and error estimation, *Engineering with Computers*, **33** (3), 607-620, (2017).

6 Rodríguez, B., Rodríguez, A.J., Sputh, B., Pastorino, R., Naya, M.Á. and González, F. Energy-based monitoring and correction to enhance the accuracy and stability of explicit co-simulation, *Multibody System Dynamics*, **55**, 103-136, (2022).

7 Andry, A.N., Shapiro, E.Y. and Chung, J. Eigenstructure assignment for linear systems, *IEEE Transactions on Aerospace and Electronic Systems*, **AES-19** (5), 711-729, (1983).

8 Zhang, J., Ouyang, H. and Yang, J. Partial eigenstructure assignment for undamped vibration systems using acceleration and displacement feedback, *Journal of Sound and Vibration*, **333** (1), 1-12, (2014).

9 Peiret, A., González, F., Kövecses, J. and Teichmann, M. Co-Simulation of Multibody Systems With Contact Using Reduced Interface Models, *Journal of Computational and Nonlinear Dynamics*, **15** (4), 041001, (2020).

10 Benedikt, M., Watzenig, D. and Hofer, A. Modelling and analysis of the non-iterative coupling process for co-simulation, *Mathematical and Computer Modelling of Dynamical Systems*, **19** (5), 451-470, (2013).

11 Belotti, R. and Richiedei, D. Pole assignment in vibrating systems with time delay: An approach embedding an a-priori stability condition based on Linear Matrix Inequality, *Mechanical Systems and Signal Processing*, **137**, 106396, (2020).

12 Liu, Z., Pang, J., Ouyang, H. and Xie, G. A new general method for assigning frequencies with low spillover through structural modifications, *Journal of Low Frequency Noise, Vibration and Active Control*, **42** (4), 1900-1918, (2023).

13 Tamellin, I., Richiedei, D., Rodríguez B. and González, F. Eigenstructure assignment and compensation of explicit co-simulation problems, *Mechanism and Machine Theory*, **176**, 105004, (2022).

14 Van Load, C.F. and Golub, G. *Matrix Computations, third ed.*, The Johns Hopkins University Press, Baltimore and London (2006).