Co-simulation benchmarks: Linear oscillator

Laboratorio de Ingeniería Mecánica - LIM

Universidade da Coruña

June 11, 2021

This document describes a classical benchmark in the co-simulation literature: a two degree-of-freedom linear oscillator.

Contact Francisco González (f.gonzalez@udc.es) if you have questions or comments about this example.

1   Problem description

This problem consists in a two-degree-of-freedom linear oscillator, shown in Fig. 1, composed by two masses m_1 and m_2 connected to each other and to the ground by means of linear springs and dampers. Variables x_1 and x_2 measure the displacement of each mass with respect to its equilibrium position, in which all the spring forces are zero. The initial position and velocity of each mass are denoted by $x_{i,0}$ and $\dot{x}_{i,0}$, respectively, with $i = 1, 2$. Similar systems have been employed as benchmark problems in the co-simulation literature, e.g., [1, 2, 3, 4, 5].
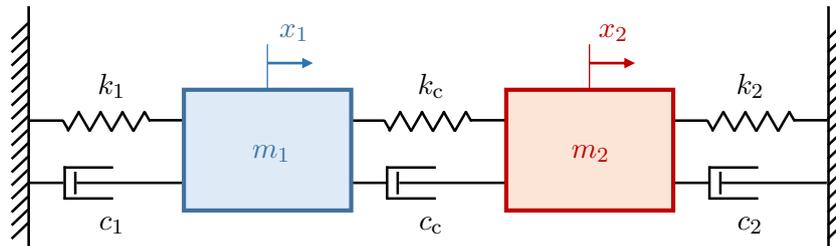
Figure 1: A two-degree-of-freedom linear oscillator

The linear oscillator problem has a known analytical solution.

1.1   Physical properties

Different values of the system parameters can be selected to represent a wide range of mechanical systems. Table 1 shows the combinations of parameters used for this benchmark problem in particular.

	m_1	m_2	k_1	k_c	k_2	c_1	c_c	c_2	$x_{1,0}$	$x_{2,0}$	$\dot{x}_{1,0}$	$\dot{x}_{2,0}$
Case	(kg)	(kg)	(N/m)	(N/m)	(N/m)	(Ns/m)	(Ns/m)	(Ns/m)	(m)	(m)	(m/s)	(m/s)
1	1	1	10	100	1000	0	0	0	0	0	100	-100
2	1	1	10	100	1000	0.01	0.01	0.01	0	0	100	-100

Table 1: Combinations of system parameters used in this problem

1.2 Dynamics equations

The oscillator dynamics is described by two ordinary differential equations (ODE) that can be expressed in terms of the mass positions $\mathbf{x} = [x_1, x_2]^{\mathrm{T}}$ and their derivatives with respect to time as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} \tag{1}$$

where

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} ; \qquad \mathbf{C} = \begin{bmatrix} c_1 + c_c & -c_c \\ -c_c & c_2 + c_c \end{bmatrix} \tag{2}$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_c & -k_c \\ -k_c & k_2 + k_c \end{bmatrix} ; \quad \mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

If the external forces $\mathbf{f}$ are zero, an analytical solution of the dynamics equations (1) can be found reordering them into the form

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} \tag{3}$$

where

$$\mathbf{z} = \begin{bmatrix} x_1 & x_2 & \dot{x}_1 & \dot{x}_2 \end{bmatrix}^{\mathrm{T}} \tag{4}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(k_1 + k_c)/m_1 & k_c/m_1 & -(c_1 + c_c)/m_1 & c_c/m_1 \\ k_c/m_2 & -(k_2 + k_c)/m_2 & c_c/m_2 & -(c_2 + c_c)/m_2 \end{bmatrix} \tag{5}$$

The analytical solution to Eq. (3) has the form

$$\mathbf{z}\left(t\right) \;=\; e^{\mathbf{A}\left(t - t_0\right)} \cdot \mathbf{z}_0 \tag{6}$$

The linear oscillator can also be simulated using a monolithic approach solving the initial value problem defined by Eq. (1) and the starting system configuration and velocity. A possibility is using the semi-implicit forward Euler formula as numerical integrator

$$\dot{\mathbf{x}}_{k+1} \;=\; \dot{\mathbf{x}}_k + h\ddot{\mathbf{x}}_k \; ; \qquad \mathbf{x}_{k+1} \;=\; \mathbf{x}_k + h\dot{\mathbf{x}}_{k+1} \tag{7}$$

where h is the integration step-size.

2 Subsystem definition

The oscillator can be decomposed into two subsystems, each of them taking care of the simulation of the dynamics of one of the point masses. Afterwards, co-simulation can be performed according to many different coupling schemes, e.g., Jacobi and Gauss-Seidel configurations, either explicit or implicit, both in single-rate and multi-rate communication grids. The following selections of coupling variables are put forward for the definition of this benchmark example:

- Force-displacement coupling (**f-s**)

- Displacement-displacement coupling (**s-s**)

- Force-force coupling (**f-f**)

The selection of coupling variables determines subsystem definition, as shown next.

2.1 Force-displacement co-simulation

A first option is dividing the oscillator into two subsystems as shown in Fig. 2. Subsystem $\mathcal{M}_1$ integrates the dynamics of the first mass and passes the coupling force f^{c} as output to subsystem $\mathcal{M}_2$, which takes care of the second mass and, in turn, delivers its position as output, ξ^{c}.
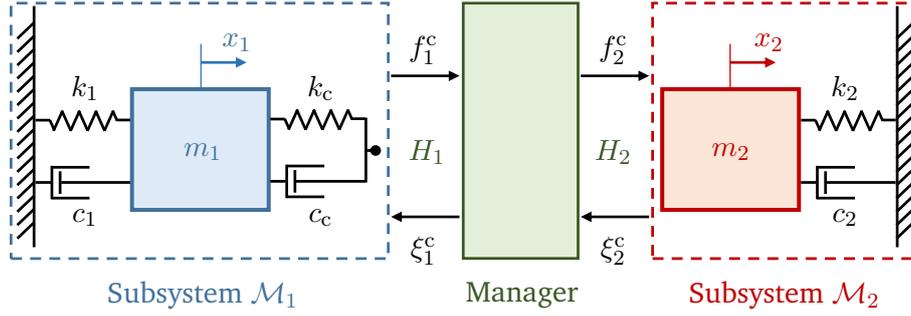
Figure 2: The linear oscillator arranged following a force-displacement coupling scheme

In principle, $f_1^c \neq f_2^c$ and $\xi_1^c \neq \xi_2^c$ because input extrapolation or other adjustments can be performed by the co-simulation manager. Each subsystem has its own internal integration step-size, h_1 and h_2; the macro step-sizes with which each subsystem communicates with the co-simulation manager are H_1 and H_2 and do not need to be the same either.

In this configuration, subsystem $\mathcal{M}_1$ has direct feed-through, because the evaluation of its output – the coupling force f^c– requires the availability of input ξ^c at any given instant in time

$$f^{\mathrm{c}} = k_{\mathrm{c}} \left(x_1 - \xi^{\mathrm{c}} \right) + c_{\mathrm{c}} \left(\dot{x}_1 - \dot{\xi}^{\mathrm{c}} \right) \tag{8}$$

The derivative $\dot{\xi}^{\mathrm{c}}$ can either be directly passed by the second subsystem as part of its output, or evaluated via numerical differentiation of ξ^{c}.

2.2 Displacement-displacement co-simulation

Another possibility is having both subsystems send their position (ξ and η) as output to the co-simulation manager, as in Fig. 3. In this case, both subsystems need access to the information about the coupling stiffness and damping parameters, k_{c} and c_{c}.

In this co-simulation configuration, none of the subsystems has direct feed-through. Subsystem output is part of the state in both cases and can be evaluated without knowledge of the input at any time. Accordingly, input extrapolation is not necessary with this configuration.
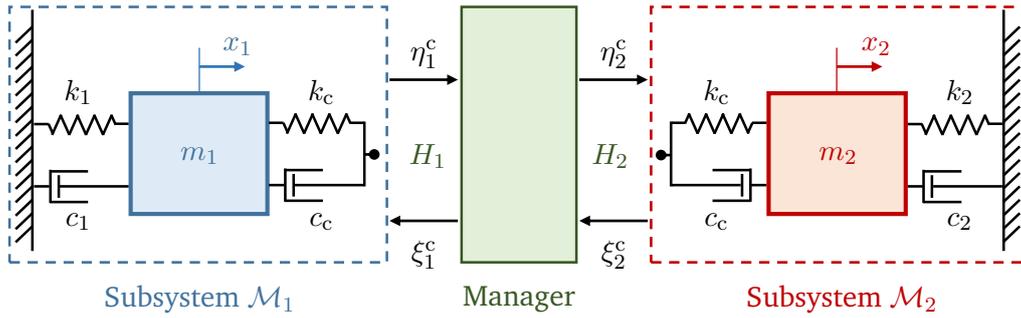
Figure 3: The linear oscillator arranged following a displacement-displacement coupling scheme

2.3 Force-force co-simulation

It is also possible to perform the evaluation of the coupling force outside the subsystems, for instance in the co-simulation manager. In this case, the subsystems would provide information about their position and velocity to the manager, and would receive the coupling force as input, as shown in Fig. 4.
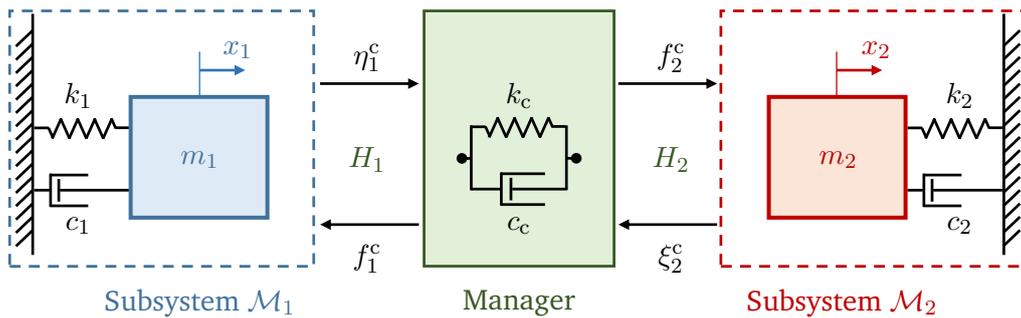
Figure 4: The linear oscillator arranged following a force-force coupling scheme

In this case, the information about the stiffness and damping properties of the coupling interface must be available to the co-simulation manager. The scheme is similar to the displacement-displacement one, in the sense that the subsystems are free from direct feed-through. In fact, for single-rate co-simulation schemes, both coupling approaches are equivalent. The need to perform input extrapolation in the multi-rate case introduces differences between their results, though.

3   Simulation

A 10-s simulation of the system motion, starting from the initial configuration and velocity, in the absence of externally applied forces, is the simulation considered in the definition of this benchmark problem.

For case 1 in Table 1 the system is conservative, and so its mechanical energy should remain constant during motion.

References

[1] González, F., Arbatani, S., Mohtat, A., Kövecses, J.: Energy-leak monitoring and correction to enhance stability in the co-simulation of mechanical systems. Mechanism and Machine Theory **131**, 172–188 (2019). DOI 10.1016/j.mechmachtheory.2018.09.007

[2] González, F., Naya, M.Á., Luaces, A., González, M.: On the effect of multirate co-simulation techniques in the efficiency and accuracy of multibody system dynamics. Multibody System Dynamics **25**(4), 461–483 (2011). DOI 10.1007/s11044-010-9234-7

[3] Sadjina, S., Kyllingstad, L.T., Skjong, S., Pedersen, E.: Energy conservation and power bonds in co-simulations: non-iterative adaptive step size control and error estimation. Engineering with Computers **33**(3), 607–620 (2017). DOI 10.1007/s00366-016-0492-8

[4] Schweizer, B., Li, P., Lu, D.: Explicit and implicit cosimulation methods: stability and convergence analysis for different solver coupling approaches. Journal of Computational and Nonlinear Dynamics **10**(5–051007), 1–12 (2015). DOI 10.1115/1.4028503

[5] Schweizer, B., Lu, D.: Semi-implicit co-simulation approach for solver coupling. Archive of Applied Mechanics **84**(12), 1739–1769 (2014). DOI 10.1007/s00419-014-0883-5